Scheme of Cooperative Spectrum Sensing Based on Adaptive Decision Fusion Algorithm

Xing-Xiong Xu, Li-Min Wu, and Wei Chen

Abstract—Spectrum sensing is one of the core technologies for cognitive radios (CR), where reliable detection of the signals of primary users (PUs) is precondition for implementing the CR systems. A cooperative spectrum sensing scheme based on an adaptive decision fusion algorithm for spectrum sensing in CR is proposed in this paper. This scheme can estimate the PU prior probability and the miss detection and false alarm probabilities of various secondary users (SU), and make the local decision with the Chair-Varshney rule so that the decisions fusion can be done for the global decision. Simulation results show that the false alarm and miss detection probabilities resulted from the proposed algorithm are significantly lower than those of the single SU, and the performance of the scheme outperforms that of the cooperative detection by using the conventional decision fusion algorithms.

Index Terms—Cognitive radio, Chair-Varshney rule, decision fusion, energy detection, spectrum sensing.

1. Introduction

It is known to us that the traditional wireless network adopts inflexible spectrum allocation policies, which lead to a large amount of vacancy within most of licensed spectra while they are partially overused in a certain period[1]. Joseph Mitola proposed the idea of cognitive radio(CR) in 1999, which was an ideal solution to the shortage problem of available spectrum, where a CR user could sense spectrum holes[3] in the spectral environment and adapt its transmission parameters to have a dynamical access to them. Thus spectrum sensing is the precondition for implementing the CR systems.

Because the single secondary user (SU) has limited sensing performance, and may not be able to reliably detect the presence of primary users (PUs) in some frequency band, especially under the influence of channel fading and shadowing and the hidden point problem[4], it can not satisfy the reliability requirement of CR systems. Cooperative spectrum sensing combines SUs distributed in various areas and uses SUs’ local decisions that are transmitted to a decision fusion center to get a final global decision[5][6], which can decrease sensing uncertainty of single SU so as to improve the sensing reliability of CR systems. The optimal decision fusion rule for a distributed detection system — Chair-Varshney rule — has been proposed in [7], where the presence probability of PU and false alarm and miss detection probabilities of SUs were required to calculate the test statistics. However, those probabilities are usually unknown or variant in reality. Thus, this paper proposes a scheme of cooperative spectrum sensing based on an adaptive decision fusion algorithm, which can estimate those parameters needed and then make the global decision by using the Chair-Varshney rule. The main points of the local sensing model and the cooperative spectrum sensing based on the adaptive decision fusion algorithm in the proposed scheme are expounded as follows.

2. Local Sensing Model

As we know, the detection of whether the PU signals are present can be stated as a binary hypothesis testing in essence. Let $s(t)$ and $y(t)$ denote the PU signal and the signal received by the SU, respectively. $n(t)$ denotes the additive white Gaussian noise (AWGN). $H_0$ and $H_1$ represent the absence and presence of the primary signal, respectively. The detection model can be expressed as

$$y(t) = s(t) + n(t)\begin{cases}H_0 & H_1 \end{cases}$$

(1)

Generally speaking, there is none of a prior knowledge of PU in this condition, but we can adopt energy detection[8] that is usually used to detect the unknown signal in the noise. And the energy detection model can be defined as

$$Y = \frac{1}{N} \sum_{n=1}^{N} |y(n)|^2 \begin{cases} \chi^2_N \lambda & H_0 \ wounds < \lambda \ H_1 \\
\chi^2_N (2\gamma) \lambda & H_1 \end{cases}$$

(2)

where $\gamma$ is the signal to noise ratio (SNR), $Y$ is the statistic value of detection, $y(n)$ is the discrete signal received by the
SU while \( n \) is the discrete time, \( \lambda \) is the detection threshold, and \( N=2TW \) is the number of samples. \( TW \) denotes the product of detection time \( T \) and signal bandwidth \( W \). \( \chi_{2TW}^2 \) represent a central chi-square distribution with \( 2TW \) degrees of freedom and \( \chi_{2TW}^2(2\gamma) \) denotes a non-central chi-square distribution with \( 2TW \) degrees of freedom and a non-central parameter \( 2\gamma \). Commonly, \( p_f \) and \( p_m \) are used to denote the false alarm probability and miss detection probability of every SU, respectively, thus, it can be derived that

\[
\begin{align*}
    p_f &= p\{Y > \lambda | H_0\} = \frac{\Gamma(TW, \lambda/2)}{\Gamma(TW)} \\
p_m &= p\{Y > \lambda | H_1\} = Q_{\text{df}}(\sqrt{2\gamma}, \sqrt{\lambda})
\end{align*}
\]

(3)

where \( \Gamma(\cdot, \cdot) \) and \( Q(\cdot) \) are incomplete gamma function and generalized Marcum Q-function, respectively. Obviously, \( p_f \) is unrelated to \( \gamma \) and thus \( \lambda \) can be calculated by setting the value of \( p_f \) according to the formulas above.

3. Cooperative Spectrum Sensing Based on the Adaptive Decision Fusion Algorithm

3.1 Distributed Parallel Cooperative Spectrum Sensing Model

The structure of distributed parallel cooperative spectrum sensing in a CR network is shown in Fig. 1. In the structure, the primary signal is emitted from the PU, then received by SUs via various channels. Let \( y_1, y_2, \cdots, y_N \) denote the signals received by the \( N \) SUs, respectively. Based on the local observation, each SU makes a local binary decision that is denoted by \( u_i \) (\( u_i = 0 \) or \( 1 \), \( i = 1, 2, \cdots, N \)), and SU forwards it to the fusion center so as to make the global decision \( u_0 \).

There are many fusion rules can be used to obtain \( u_0 \) such as AND rule, OR rule, and KN rule (that is Majority rule). The AND rule can significantly diminish the false alarm probability of the CR network at the cost of a big diminution of the detection probability, and the OR rule is just the reverse. The KN rule is a compromise between the AND rule and OR rule, which can improve the detection probability as much as possible, and meet the limitation of the false alarm probability at the same time. Although those three rules above are simple, they are not practical for ignoring the presence probability of the binary hypothesis and the probability characteristics of various SUs.

The Chair-Varshney rule proposed in [7] is an optimal fusion rule for a distributed parallel detection system. In a CR network that consists of \( N \) SUs, its statistic decision value can be depicted as the sum of weighted local decisions:

\[
\omega_h + \sum_{i=1}^{N} \omega_i u_i > 0
\]

(4)

Formula (4) means if \( \omega_h + \sum_{i=1}^{N} \omega_i > 0 \) hypothesis \( H_1 \) is true, while if \( \omega_h + \sum_{i=1}^{N} \omega_i < 0 \) hypothesis \( H_0 \) is true. In (4), \( \omega_h \) and \( \omega_i \) are weight factors, and they can be expressed as

\[
\omega_h = \log\left(\frac{P_1^0}{P_0^h}\right)
\]

(5)

\[
\omega_i = \begin{cases} 
\log\left(\frac{1 - P_m^i}{P_f}\right), & u_i = 1 \\
\log\left(\frac{P_m^i}{1 - P_f}\right), & u_i = 0 
\end{cases} \quad (i = 1, 2, \cdots, N)
\]

(6)

where \( P_0 = p(H_0) \) is the absence probability of PU, \( P_1 = p(H_1) = 1 - P_0 \) is the relevant presence probability, and \( P_m^i \) and \( P_f^i \) are the false alarm and miss detection probabilities of the \( i \)th SU, respectively. Obviously, every weight factor is the function of the false alarm and miss detection probability of the relevant SU. Thus, before doing decision fusion, we must calculate the values of parameters \( P_1 \), \( P_f^i \), and \( P_m^i \) that are unknown or variant in reality. So it is needed to take the adaptive decision fusion algorithm into consideration, as it can approximately calculate those parameters mentioned above.

3.2 Adaptive Decision Fusion Algorithm

In a CR network that is comprised of three SUs, where local decisions made by SUs are assumed to be independent statistically, we can estimate the unknown probabilities \( P_1 \), \( P_f^i \), and \( P_m^i \) \((i = 1, 2, 3)\) by each local decision’s unconditional probability and their joint probability. Let \( \nu_{u_1, u_2, u_3} \) denote the joint probability of \( u_1, u_2, u_3 \) \((u_1, u_2, u_3 = 0 \) or \( 1) \). According to the theorem in [9], if a distributed parallel cooperative spectrum sensing system satisfies

\[
0 < P_1 < 1, \quad P_f^i + P_m^i < 1, \quad (i = 1, 2, 3)
\]

we can estimate \( P_1 \), \( P_f^i \), and \( P_m^i \) by \( (8) \) to \( (14)[9] \):

\[
P_i = 0.5 - \frac{X}{2\sqrt{X^2 + 4}}
\]

(8)

\[
P_0 = 1 - P_1
\]

(9)

Fig. 1. Distributed parallel cooperative spectrum sensing structure.
\[ P_{ij} = \gamma_i - a \frac{P_i}{\sqrt{1-P_i}} \]  
\[ P_{i}^* = 1 - \gamma_i - a \frac{1-P_i}{\sqrt{1-P_i}} \]

where

\[ X = y_1 - \gamma y_2 - y_3 = y_1 y_2 + y_3 + y_4 a_1 a_2 a_3 \]

\[ \sqrt{\delta_1 - y_1 y_2} (\delta_1 - y_1 y_3) (\delta_1 - y_2 y_3) \]

\[ y_i = \sum \frac{P_{iu,y_1}}{P_i} y_2 = \sum \frac{P_{iu,y_2}}{P_i} y_3 = \sum \frac{P_{iu,y_3}}{P_i} y_4 = P_{111} \]

\[ \delta_{12} = P_{110} + P_{111}, \delta_{23} = P_{110} + P_{111}, \delta_{32} = P_{110} + P_{011} \]

\[ \begin{align*}
    a_1 &= \sqrt{(\delta_2 - y_1 y_2)(\delta_3 - y_1 y_3)(\delta_3 - y_2 y_3)} \\
    a_2 &= \sqrt{(\delta_2 - y_1 y_2)(\delta_2 - y_2 y_3)(\delta_3 - y_1 y_3)} \\
    a_3 &= \sqrt{(\delta_2 - y_1 y_2)(\delta_3 - y_2 y_3)(\delta_2 - y_1 y_3)}
\end{align*} \]

The theorem indicates that, if \( \gamma \) and \( \delta \) can be approximated by averaging the local decision values as

\[ \hat{\gamma}_i = \frac{1}{T} \sum_{t=1}^{T} u_t^{i} \]

where \( u_t^{i} \) is the decision made by the \( i \)th SU at the time point \( t \), and \( T \) is the duration time. Equation (15) can also be expressed in the iterative form as

\[ \hat{\gamma}_i = \frac{1}{T} u_t^{i} + \frac{T-1}{T} \hat{\gamma}_{i-1} = \hat{\gamma}_{i-1} + \frac{1}{T} (u_t^{i} - \hat{\gamma}_{i-1}). \]

Similarly, \( \hat{\gamma}_j \) and \( \hat{\delta}_{j,k} \) are obtained by

\[ \hat{\gamma}_j = \frac{1}{T} u_t^{j} + \frac{T-1}{T} \hat{\gamma}_{j-1} \]

\[ \hat{\delta}_{j,k} = \frac{1}{T} u_t^{k} u_t^{2} + \frac{T-1}{T} \hat{\delta}_{k-1} \]

where \( \hat{\gamma}_i \) and \( \hat{\gamma}_j \) can be any initial value. Then, substituting these estimation values into (8) to (14), \( P_1, P_2, P_3, P_4, P_5 \), and \( P_6 \) (i=1, 2, 3) can be obtained.

In a CR network that is comprised of \( N \) SUs, the relation of the false alarm probability and the miss detection probability between the \( i \)th and the \( j \)th (i, l=1,2,...,N; i\neq l) SU satisfies the following, respectively.

\[ P_{ij} = \frac{P_{ij} \delta_{ij}}{(1-P_i) + \gamma_{ij} P_{ij}} + \gamma_{ij} P_{ij} \]

\[ P_{i}^* = 1 + \frac{(\gamma_{i} y_{i} - (1-P_i) \delta_{i}}{(1-P_i) + \gamma_{i} (1-P_{i}^*)} \]

According to the analysis above, the adaptive decision fusion algorithm can be concluded as follows.

1) Initialization: choosing the initial values of \( \gamma_{i,j} \) (i=1,2,3), \( \delta_{j,k} \) (j,k=1,2,3; j\neq k).

2) Iteration:
   A. Calculate parameters of three SUs.
   a) Substitute \( u_t^{i} , u_t^{j} , \) and \( u_t^{k} \) into (15) to (18) to update \( \hat{\gamma}_i \) (i=1,2,3), \( \hat{\gamma}_j \) and \( \hat{\delta}_{j,k} \) (j,k=1,2,3; j\neq k).
   b) Substitute updated \( \hat{\gamma}_i , \hat{\gamma}_j , \) and \( \hat{\delta}_{j,k} \) into (12) to (15) to calculate \( \hat{\gamma}_i \) and \( \hat{\delta}_{j,k} \) (i=1,2,3), where \( \hat{\gamma}_i \) is the estimated value of \( \gamma_i \) in (12) and \( \hat{\delta}_{j,k} \) is the estimated value of \( \delta_{j,k} \) in (14), then compute \( \hat{\gamma}_i \) and \( \hat{\gamma}_j \) using the following equations:

\[ \hat{\gamma}_i = \bigg[ 0.5 - \hat{\gamma}_i \bigg] \bigg[ 2 \sqrt{\hat{\gamma}_i} + 4 \bigg] \]

\[ \hat{\gamma}_j = \bigg[ \frac{1}{1 - \hat{\gamma}_j} \bigg] \bigg[ \frac{1}{1 - \hat{\gamma}_j} \bigg] \]

\[ \hat{\delta}_{j,k} = \bigg[ 1 - \hat{\gamma}_j - \frac{1}{\hat{\gamma}_j} \bigg] \bigg[ \frac{1}{1 - \hat{\gamma}_j} \bigg] \]

Equations (21) to (24) are the transformations of (8) to (11). \( \bigg[ \bigg] \) means the conversion of mapping to unit interval, and \( \bigg[ \bigg] \) is the maximum of \( \min \{1, \Re(z) \} \) for \( \Re \in C \).

B. Use \( u_t^{i} \) to update \( \hat{\gamma}_i \) (i=4,5,...,N) and \( \hat{\delta}_{j,k} \) (k=4,5,...,N) according to (15) and (16).

C. Use updated parameters of b) in Step A and \( \hat{\gamma}_i \) and \( \hat{\delta}_{j,k} \) (i=1,2,3) to calculate \( \hat{\gamma}_i \) and \( \hat{\gamma}_j \) according to (19) and (20).

D. Substitute \( \hat{\gamma}_i \), \( \hat{\gamma}_j \), \( \hat{\gamma}_j \) and \( \hat{\gamma}_j \) (i=1,2,...,N) into (4) to (6) to obtain the optimal decision weights \( \hat{\omega}_b \) and \( \hat{\omega}_b \) (i=1,2,...,N).

Since the update rules (15) to (18) are iterative, we only need the next local decision to obtain the updated values after fixing the iterations. Thus the algorithm needs fixed memory space at all times and the calculation amount does not increase with iterations increasing, which is one of the most significant advantages in the proposed algorithm.

4. Simulation Results and Discussion

4.1 Validity of Adaptive Decision Fusion Algorithm

Simulations are performed for a CR network comprised of \( N \) SUs, setting \( N=3 \). A binary phase shift keying (BPSK) signal is used for the PU signal. And the energy detection scheme is employed for sensing the spectrum of PU in the AWGN channel by using the Monte Carlo simulation method. Assume \( T_m \) to be simulation times, and let \( T_w=20000 \). Assume \( P(H_i)=0.7 \), false alarm and miss detection probabilities of the three secondary users are \( P_{ij} = 0.002 \), \( P_{j} = 0.10 \), \( P_{ij} = 0.035 \), \( P_{j} = 0.012 \),
The scheme for sensing secondary users in a CR network is based on adaptive decision fusion and the single secondary user scheme. We set $\{\gamma_{0}\} = \{3, 5, 2, -2, 3, -1, -3, 1, 0\}$ in dB and $\{P_{0}\} = \{0.02, 0.04, 0.07, 0.1, 0.05, 0.5, 0.2, 0.3, 0.6\}$. The Chair-Varshney rule and traditional fusion rules such as AND rule, OR rule, and KN rule are adopted in simulation. The KN rule also called “K out of N” rule means the global decision will be $H_{1}$, if K or more users choose $H_{1}$. And usually, let $K = N/2$. From Fig. 4 (a), it is shown that the false alarm probability of the proposed scheme is 23.9 times and 719.7 times lower than that of the best KN rule and OR rule, respectively. Similarly, Fig. 4 (b) shows that the miss detection probability of the proposed scheme is 8.7 times and 87.1 times lower than that of the best KN rule and AND rule, respectively. Simulation results indicate that the performance of the proposed scheme outperforms that of the cooperative detection by using the conventional data fusion algorithms.

As shown in Fig. 2, it comes to a conclusion that as the simulation times increase, the estimate values of false alarm and miss detection probabilities obtained by the adaptive decision fusion algorithm finally converge to their real values, respectively, which verifies the validity of the algorithm.

4.2 Comparison between Proposed Spectrum Sensing Scheme and Single Secondary User

In a CR network with $N$ secondary users, where $N=6$, $\{\gamma_{0}\}$ represents the local SNRs of these six secondary users, and we set $\{\gamma_{0}\} = \{3, 5, 2, -2, 3, -1\}$ in dB. Fig. 3 depicts the comparison of performance between cooperative spectrum sensing based on adaptive decision fusion and the single secondary user. From Fig. 3, it can be observed that the false alarm and miss detection probabilities of the proposed scheme and each secondary user are gradually leveling off with the increase of simulation times. The false alarm probability and miss detection probability of the proposed scheme is 12.8 dB and 11.8 dB, which are lower than that of the best one of the six secondary users. It indicates that the performance of the proposed scheme is superior to that of each single secondary user.

4.3 Comparison between Proposed Scheme and Scheme Based on Traditional Fusion Rules

Assume $N=9$, and various SNRs and false alarm probabilities are used to describe those secondary users in different states of channels. $\{\gamma_{0}\}$ and $\{P_{0}\}$ represent SNRs and false alarm probabilities of the nine secondary users in order, respectively. We set $\{\gamma_{0}\} = \{3, 5, 2, -2, 3, -1, -3, 1, 0\}$ in dB and $\{P_{0}\} = \{0.02, 0.04, 0.07, 0.1, 0.05, 0.5, 0.2, 0.3, 0.6\}$. The Chair-Varshney rule and traditional fusion rules such as AND rule, OR rule, and KN rule are adopted in simulation. The KN rule also called “K out of N” rule means the global decision will be $H_{1}$, if K or more users choose $H_{1}$. And usually, let $K = N/2$. From Fig. 4 (a), it is shown that the false alarm probability of the proposed scheme is 23.9 times and 719.7 times lower than that of the best KN rule and OR rule, respectively. Similarly, Fig. 4 (b) shows that the miss detection probability of the proposed scheme is 8.7 times and 87.1 times lower than that of the best KN rule and AND rule, respectively. Simulation results indicate that the performance of the proposed scheme outperforms that of the cooperative detection by using the conventional data fusion algorithms.
Fig. 4. Comparison of cooperative spectrum sensing under adaptive decision fusion and traditional fusion rules: (a) false alarm probabilities under adaptive decision fusion and other fusion rules and (b) miss detection probabilities under adaptive decision fusion and other fusion rules.

5. Conclusions

Single PU lacks sensing reliability due to its limited detection ability, while cooperative spectrum sensing can effectively improve the reliability of detection. But the implementation of the optimal fusion rule for distributed parallel cooperative detection—the Chair-Varshney rule—needs the presence probability of PU and false alarm and miss detection probabilities of various SUs, while those parameters are unknown or variant in reality. This paper proposes a scheme of cooperative spectrum sensing based on the adaptive decision fusion algorithm, where the fusion center uses each secondary users’ local decision to estimate those parameters needed, and makes the global decision by using the Chair-Varshney rule. Simulation results show that the performance of adaptive decision fusion based cooperative spectrum sensing is significantly better than that of the single SU, and has some merits compared with the traditional fusion rules based cooperative spectrum sensing as well.

References


