Beam Pattern Design and Performance Analyses for Circular Array with Spherical Baffles

Cheng Zhang, Ke-An Chen, and Guo-Yue Chen

Abstract—The influence of a rigid spherical baffle on the response of a uniform circular microphone array (UCA) is analyzed and two eigen-beam beamforming arrays are designed in the eigen-beam subspace derived from the soundfield decomposition. Expressions of white noise gain (WNG) and directivity index (DI) are derived for the designed arrays. Performance analyses are carried out for the designed arrays and compared between those of the delay-and-sum beamforming array using UCA with and without a rigid sphere. Computer simulations demonstrate that the designed arrays have frequency-independent directivity with the cost of reduced robustness at low frequency band. The delay-and-sum beamforming array has constant WNG at all frequencies, while its directivity of which is reduced at low frequency band. The rigid sphere can improve the robustness for all the arrays.

Index Terms—Beamforming, directivity index, uniform circular array, white noise gain.

1. Introduction

An audition system can provide a robot with the ability to receive and process sounds arriving from any direction, thus enhance the sensory information about its local environment. Many array signal processing algorithms for robots audition system have been investigated in the literatures\cite{1,2}. The delay-and-sum beamforming method is a technique for forming a strong directivity in the direction of the purpose and giving a high robustness, which is used in audition system of robot in \cite{3}. Most of these algorithms, however, deal with arrays in free field. In practice, arrays must be located on some objects which may cause some effects on the response of arrays to received signals, e.g. the body especially the head of a robot will take some effects on the soundfield around the array\cite{4,5}. In present literatures, spherical arrays gradually become a subject of interest since three dimensional sampling of the soundfield is allowed and the beam can be steered to any 3D direction with the same beam pattern\cite{6}. But the number of elements used in spherical arrays is usually large in order to get higher order of spherical harmonics, and the positions of elements are specifically laid out on the surface of the sphere needed by the orthogonal sampling, which limits the engineering implementation of spherical arrays. The next selection of array setup with baffles is uniform circular array (UCA)\cite{7,8}. An algorithm named EB-ESPRIT (Eigen-beam ESPRIT) was developed using a UCA mounted into a cylindrical baffle in \cite{7}, which can deal with multiple wideband acoustic sources effectively. In \cite{8}, broadband beam pattern design was studied for UCA using monopole and dipole sensors respectively taking into account of the scattering soundfield of a rigid sphere.

This paper studies the design and performance analyses of two kinds of beamforming arrays for a circular array mounted on a rigid spherical baffle. The influence of the rigid sphere on the response of the circular array is analyzed. Then, two eigen-beam beamforming arrays are designed. Expressions of white noise gain (WNG) and directivity index (DI) of the designed arrays are derived after some algebraic manipulations. Performance analyses for the designed arrays and the delay-and-sum array are carried out using WNG and DI as the evaluation metric. Finally, computer simulations are given.

2. Soundfields Decomposition

Assuming a UCA with radius $R_0$ is mounted around a rigid sphere with radius $R$, and a unit magnitude impinging plane wave comes from the direction $(\theta, \phi_0)$, the equation describing the resulting sound pressure in spherical coordinates is $g(kr) e^{-jkr}$

$$g = \sum_{n=0}^{\infty} \left(2n+1\right)(-j)^n \left[ h_n(kR_0) - h_n(kR) \right]$$

$$+ \sum_{n=0}^{\infty} \frac{(n-|\theta|)!}{(n+|\theta|)!} \gamma_n \beta_n \left| \cos \theta \right| e^{-j\phi_0} (1)$$

where $k$ is the wave number, $k = 2\pi/\lambda$, $\lambda$ is the wave length of the plane wave, $j_n(\cdot)$ is the $n$th-order spherical
Bessel function of the first kind, \( b_n(\cdot) \) is the \( n \)-th-order spherical Hankel function, \( j_n(\cdot) \) and \( h_n(\cdot) \) denote the derivative of the \( n \)-th-order spherical Bessel function and the \( n \)-th-order spherical Hankel function with respect to the argument respectively, \( P_n^{\phi}(\cdot) \) is the Legendre function of order \( n \) and degree \( q \). The time dependency is omitted in the equation.

For simplicity, the theoretical construct of a continuous circular aperture and a unit magnitude plane wave impinging from \( \theta = \pi/2 \) are concerned firstly. Assuming the continuous circular aperture is located on the equator of the rigid sphere, i.e. \( \theta = \pi/2 \), and the sound pressure at the position of the continuous circular array is described by \( r = R \), then, (1) becomes

\[
g = \sum_{n=0}^{\infty} \left\{ \frac{(-1)^n}{(n+1)!} \left[ j_n(kR) - \frac{j_n'(kR_0)}{h_n(kR_0)} h_n(kR) \right] \right. \\
\left. \times \left( \frac{n-|p|}{n+|p|} \right) \sum_{q=-\infty}^{\infty} \left( \frac{2q}{n+q} \right) \phi^{q\theta} \phi^q \right\}
\]

(2)

where \( g \) can be seen as the transfer function from the source point \((\theta_0, \phi_0)\) to the sensor location \((R, \theta, \phi)\).

Applying soundfield decomposition theory, the \( p \)-th order eigen-beam derived from the decomposed soundfield can be described as [8]

\[
P_p = b_p e^{jp\theta_0}
\]

(3)

where

\[
b_p = \sum_{n=0}^{\infty} \left\{ \frac{(-1)^n}{(n+1)!} \left[ j_n(kR) - \frac{j_n'(kR_0)}{h_n(kR_0)} h_n(kR) \right] \right. \\
\left. \times \left( \frac{n-|p|}{n+|p|} \right) \sum_{q=-\infty}^{\infty} \left( \frac{2q}{n+q} \right) \phi^{q\theta} \phi^q \right\}
\]

(4)

\( P_p \) can be seen as the coefficient of the Fourier transform for the pressure and it can also be referred to as the eigen-beam. The highest order of the eigen-beams \( N \) is satisfied by \( N \leq [kR] \), and \([\cdot]\) stands for the biggest integer [10].

Fig. 1 shows the eigen-beam magnitude response at different frequencies.

Fig. 1. Magnitude response as a function of the orders.

Sampling the continuous circular array using a UCA results in the same expressions as (3) when the number of the elements of UCA satisfies \( M \geq 2N \) [10].

### 3. Beam Pattern Design

Considering a square integrable function on the unite circle, this function can be expressed as the summation of Fourier series

\[
f(\varphi) = \sum_{p=-\infty}^{\infty} F_p e^{jp\varphi}.
\]

(5)

The coefficients of Fourier series is defined as

\[
F_p = \frac{1}{2\pi} \int_{0}^{2\pi} f(\varphi) e^{-jp\varphi} d\varphi.
\]

(6)

Sampling the pressure on the rigid spherical scatterer using circular array, the array output can be calculated as

\[
y = \frac{1}{2\pi} \sum_{p=-\infty}^{\infty} P_p w^*_p(\varphi) d\varphi
\]

(7)

where \( w(\varphi) \) is the array weighting function, \(*\) denotes the complex conjugate. Because \( P(\varphi) \) and \( w(\varphi) \) are square integrable, \( \{P(\varphi), P_p\} \) and \( \{w(\varphi), w_p\} \) can be seen as two Fourier transform pairs. Then (7) can be expressed as

\[
y = \sum_{p=-\infty}^{\infty} P_p w^*_p.
\]

(8)

The array weights can be generally written as [11]

\[
w_p = \frac{d_p}{b_p} e^{jp\varphi}
\]

(9)

\[
w(\varphi) = \sum_{p=-\infty}^{\infty} \left( \frac{d_p}{b_p} e^{jp\varphi} \right) e^{jp\varphi}
\]

(10)

where \( d_p \) is a design coefficient of the weighting function. From (3), (8), and (9), the array output becomes

\[
y = \sum_{p=-\infty}^{\infty} b_p e^{jp\varphi} d_p e^{-jp\varphi}
\]

(11)

As indicated in Fig. 1, there are only finite eigen-beams with considerable magnitude response. Assuming the highest order of eigen-beams is \( N \), then (11) becomes

\[
y = \sum_{p=-N}^{N} d_p e^{-jp(\varphi-\theta_0)}.
\]

(12)
Considering a UCA of 12 omnidirectional microphones, the highest order of the eigen-beams derived from the decomposed soundfield is 6. The eigen-beams under \( N = 5 \) are used to form the beam pattern in array design. Different beam patterns can be derived by designing different values of \( d_p \). Two beam patterns are designed to investigate the characteristics of eigen-beam beamforming arrays with \( d_p \) defined respectively as: 1) \( d_0 = d_1 = \cdots = d_5 = 1, \) \( d_6 = 4.02, \) \( d_7 = 3.81, \) \( d_8 = 3.24, \) \( d_9 = 2.43, \) \( d_{10} = 1.57, \) \( d_{11} = 1. \) When \( p \leq 0 \), the following relationship is applied: \( d_p = d_p^\text{pd} \). The values of \( d_p \) for the second array are calculated using the side-lobe level controlling technique presented in [12]. The beam patterns of the designed arrays are illustrated in Fig. 2, where the EBF 1 and EBF 2 stand for the first and the second designed eigen-beam beamforming array, respectively. The first array can be seen as the plane wave decomposition array whose side-lobe level is confined under \(-30\, \text{dB}\) as shown in Fig. 2.

### 4. Performance Analysis

An important measure of array performance is the white noise gain (WNG), which evaluates the robustness of the array[13]. The WNG can be expressed as

\[
\text{WNG} = \frac{\text{w}^H \text{g}_1 \text{g}_1^H \text{w}}{\text{w}^H \text{w}}
\]  

where

\[
\text{w} = (w_1, w_2, \cdots, w_N)^T
\]

\[
w_n = \sum_{p=-\infty}^{\infty} \left( \frac{d_p^2}{b_p} e^{j2\pi m} \right) e^{j2\pi n m}
\]

(15)

where \( m = 1, 2, \cdots, M \), and \( g_i \) is the soundfield transfer function when the array look direction is \( \varphi_i \). \((\cdot)^T\) stands for the transpose, and \((\cdot)^H\) stands for the Hermitian transpose. Using (2) and (15), one can obtain

\[
\text{w}^H \text{w} = M \sum_{p=-N}^{N} \left| \frac{d_p}{b_p} \right|^2
\]

(16)

\[
\text{w}^H \text{g}_i = M \sum_{p=-N}^{N} d_p.
\]

(17)

The orthogonality property of the Fourier transform is used in the calculations. Then, (13) becomes

\[
\text{WNG} = M \left| \sum_{p=-N}^{N} d_p \right|^2 \left/ \left( \sum_{p=-N}^{N} \left| d_p \right|^2 \right) \right.
\]

(18)

Here consider two circumstances. In the first case, \( N \leq kR \), \( b_p \) is approximately constant (see Fig. 1) and can be replaced by a constant \( b \). Therefore, (18) becomes

\[
\text{WNG} \approx M \left| b \right|^2 \left/ \left( \sum_{p=-N}^{N} \left| d_p \right|^2 \right) \right.
\]

(19)

It is clear that the WNG mainly depends on \( d_p \). The WNG is larger if more eigen-beams are used in the array design as long as \( d_p \) are real. This can be explained that more eigen-beams can pick up more energy from the soundfield. On the other hand, the energy of the soundfield is distributed over more eigen-beams as frequencies increase, and \( b \) will reduce. Therefore, the WNG will decrease with frequency increasing if the same number of eigen-beams are used to form the designed beam pattern.

In the second case, \( N > kR + 1 \), which is especially important at low frequencies, because only few eigen-beams have considerable magnitude response and more weak eigen-beams must be used to form the designed beam patterns. The summation in the denominator of (18) is approximately equal to the term with the highest order when \( \left| b_{p,1} \right|^2 \ll \left| b_p \right|^2 \) in this case (see Fig. 1). So

\[
\sum_{p=-N}^{N} \left| d_p \right|^2 \approx 2 \left| \frac{d_N}{b_N} \right|^2
\]

(20)

\[
\text{WNG} \approx \frac{M}{2} \left| \frac{b_N}{d_N} \right|^2 \left/ \left( \sum_{p=-N}^{N} \left| d_p \right|^2 \right) \right.
\]

(21)

It is clearly that the WNG only depends on the highest order of \( b_p \), i.e. \( b_N \), as long as \( d_p \) are equal.

Another widely used performance measure of array is the directivity factor, which evaluates the improved directivity of the array compared to an omnidirectional microphone[13]. It can be expressed as the ratio of the array output in the look direction and the array output integrated over all directions, that is,
The summation in (27) is approximately constant, which denotes that the WNG of the delay-and-sum array is constant throughout the frequency band. In other words, the delay-and-sum array has frequency-independent robustness. Moreover, The DI of this array depends on the frequency, for $b_p$ depends on the frequency.

5. Simulation Analysis

In simulations, a UCA of twelve omnidirectional microphones with radius $R=0.116$ m is mounted around a rigid sphere with radius $R_b=0.108$. A unit magnitude plane wave arrives from $0^\circ$. The beam patterns of the designed arrays and the delay-and-sum array with $kR=0.4, 2.3, 5.1$ are shown in Fig. 3 to Fig. 5 respectively, where the DSBF stands for the delay-and-sum beamforming array. It is obvious that the beam patterns of the designed arrays have frequency-independent characteristic, and the side-lobe level of the second designed array is reduced under $-30$ dB validly. The directivity of the delay-and-sum array is very weak at low frequency ($kR=0.4$) because the values of $b_p$ in this case are very small. It increases gradually when the frequency increase ($kR=2.3$). Approximately the same directivity is derived for the delay-and-sum array and the designed arrays at high frequency ($kR=5.1$), because the values of $b_p$ are almost equal for different eigen-beams.

![Fig. 3. Beam patterns for different arrays ($kR=0.4$).](image1)

![Fig. 4. Beam patterns for different arrays ($kR=2.3$).](image2)
The DI of the designed arrays is compared with that of the delay-and-sum array for different frequencies in Fig. 6. It is shown that the DI of the designed arrays remains constant throughout the frequency band, which is a benefit for processing low frequency signals. The DI of the second designed array is somewhat lower than that of the first one, which is caused by lowering the side-lobe level. The delay-and-sum array has approximately the same DI with the designed arrays only at higher frequency band. As frequency is lowered, the DI is also reduced until the array becomes omidirectional, i.e., DI=0.

Fig. 7 shows the WNG of these arrays at different frequencies. It is clear that the delay-and-sum array has the same WNG at all frequencies, which denotes that it has good robustness. The designed arrays have approximately the same WNG with the delay-and-sum array only at higher frequencies. The WNG reduces as the frequency lowered, so the higher directivity of the designed arrays is achieved at the cost of reduced robustness. At higher frequency band, the WNG reduces when the frequency rises higher, which is consistent with the result of theoretical analysis.

The DI and WNG of the first designed array and the delay-and-sum array with and without a rigid sphere are shown in Fig. 8 and Fig. 9, respectively. Fig. 8 shows that the directivity of the former is not affected by the baffle, while that of the latter is improved somewhat. Fig. 9 shows that both of the WNG of the two arrays are improved when there is a rigid sphere, which denotes that the baffle can improve the robustness of the arrays.

6. Conclusions

It has great significance for considering the effect of array baffles when robot audition systems are designed. A rigid sphere is selected as the array baffle and its influence on the response of a UCA is analyzed. Two eigen-beam beamforming arrays are designed and their WNG and DI are compared with those of the delay-and-sum array. Computer simulations demonstrate that the designed arrays have frequency-independent directivity and the baffles can improve the robustness of the arrays. Because the delay-and-sum array has good robustness, it can be taken as
a criterion for array design. So the results derived in this paper provide a theoretical reference for array design in humanoid robot audition systems.

References


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