Design of Dynamic Controller for Neutral Differential Systems with Multiple Delays in Control Input

Zi-Xin Liu, Shu Lü, and Shou-Ming Zhong

Abstract—The design problem of dynamic output feedback controller for a class of multi-delayed neutral systems has been considered. A criterion for the existence and asymptotic stability of such controller is derived via defining a new integral operator. The criterion is expressed in terms of the linear matrix inequalities (LMI), which can be checked numerically using the effective LMI toolbox in MATLAB. One numerical example is given to illustrate the proposed design method. Numerical simulation shows that the new design method is valid.

Index Terms—Asymptotic stability, dynamic controller, Lyapunov method, time delay.

1. Introduction

In dynamic systems, there often occur time-delays, such as chemical systems, electrical networks, biological systems, economy and other disciplines. Since time delay is one of important sources of instability and poor performance, considerable efforts had been done on different aspects of delayed system \cite{1}-\cite{6}. In recent years, more attentions are focused on the stability analysis for delayed neutral differential systems \cite{7}-\cite{13}. However, the works in the above literatures are restricted to the static state feedback control schemes. Although output measurement based on static control is a necessary prerequisite for practical control problems, in some situations, there is a strong need to construct dynamic controller instead of static controller in order to obtain better performance and dynamical behavior of state response. To the best of our knowledge, the topic of dynamic output feedback control for multi-delayed neutral systems in control input has not been investigated.

Therefore, the main aim of this paper is to study the design problem of dynamic output feedback controller for a class of multi-delayed neutral systems in control input. By employing the appropriate Lyapunov functional and linear matrix inequality (LMI) technique, a criterion for the existence and asymptotic stability of the controller is derived. The criterion obtained in our paper generalizes some previous results, which will be shown by a numerical example provided later.

2. Notations and Preliminaries

Throughout this paper, $\mathbb{R}^n$ denotes $n$-dimensional Euclidean space; $\mathbb{R}^{m \times n}$ is the set of all $m \times n$ real matrices; $I$ denotes identity matrix of appropriate dimensions; $*$ represents the elements below the main diagonal of a symmetric block matrix; $|| \cdot ||$ denotes Euclidean norm of a given vector and its induced norm of a matrix; $\lambda_{\min} (\cdot)$ and $\rho(\cdot)$ denote the minimum eigenvalue and spectral radius of a matrix respectively; and $\text{diag} \{ \cdot \}$ denotes the block diagonal matrix. The notation $W > 0$ or $W \geq 0$ means that matrix $W$ is a symmetric positive definite (positive semi-definite, negative, negative semi-definite).

We consider the following multi-delayed neutral dynamical system

$$
\begin{align}
\frac{dx_i(t)}{dt} &= \sum_{j=1}^{n} a_{ij}x_j(t) + \sum_{j=1}^{n} e_{ij} \frac{dx_i(t-h)}{dt} \\
&+ \sum_{j=1}^{n} b_{i0}^ju_j(t) + \sum_{j=1}^{n} b_{i1}^ju_j(t-h) \\
y_i(t) &= \sum_{j=1}^{n} c_{ij}x_j(t), \quad i, j = 1, 2, \ldots, n
\end{align}
$$

or

$$
\begin{align}
\frac{dx(t)}{dt} &= Ax(t) + E \frac{dx(t-h)}{dt} + Bu(t) + B_0u(t-h) \\
y(t) &= Cx(t)
\end{align}
$$

with the initial condition given by

$$
x(t_{0} + \theta) = \phi(\theta), \forall \theta \in [-h', 0]
$$

where $h' = \max_{1 \leq i \leq n} \{ h_i \}$ and $h_i (i = 1, 2, \ldots, n)$ denote the transmission delays; $x(t) \in \mathbb{R}^n$ is the state vector; $A = (a_{ij})_{n \times n}$, $E = (e_{ij})_{n \times n}$, $B = (b_{ij})_{n \times m}$.
\[ B_{(t)} = (b^{(t)})_{(\text{new})} \quad \text{and} \quad C = (c^{(t)})_{(\text{new})} \in \mathbb{R}^{\text{new}} \] are constant matrices respectively; \( u(t) \in \mathbb{R}^n \) is the control input and
\[ \frac{dx(t-h) - x(t)}{dt} = \left( \frac{dx_1(t-h) - x_1(t)}{dt}, \frac{dx_2(t-h) - x_2(t)}{dt}, \ldots, \frac{dx_n(t-h) - x_n(t)}{dt} \right)^T. \]
\[ u(t-h) = (u_1(t-h_1), u_2(t-h_2), \ldots, u_n(t-h_n))^T; \]
\[ y(t) \in \mathbb{R}^n \] is the measured output, and \( \phi(t) \in \mathbb{R}^n \) is the initial vector function, where \( \phi_0 \) is a set of all continuous differentiable function on \([-h, 0)\) to \(\mathbb{R}^n\).

For stabilizing system (1), we consider the following dynamic output feedback controller
\[ \frac{\zeta(t)}{dt} = A_\xi \zeta(t) + B_\xi y(t) \quad \text{and} \quad u(t) = C_\xi \zeta(t) \tag{3} \]
where \( \zeta(t) \in \mathbb{R}^n \) is the controller state vector, \( A_\xi, B_\xi, \) and \( C_\xi \) are given matrices with appropriate dimensions needing to be determined. From (1) and (3) we can obtain the following closed-loop system
\[ \frac{dz(t)}{dt} = \bar{A}_\xi z(t) + \bar{A}_\xi z(t-h) + \bar{A}_\xi \frac{dz(t-h)}{dt} \tag{4} \]
where
\[ \bar{A}_\xi = \begin{pmatrix} A & B & C \\ B & C & A \end{pmatrix}, \quad z(t) = \begin{pmatrix} x(t) \\ \zeta(t) \end{pmatrix}, \quad B = \begin{pmatrix} 0 & B & C \\ 0 & 0 & 0 \end{pmatrix}, \quad \bar{A}_\xi = \begin{pmatrix} E & 0 \\ 0 & 0 \end{pmatrix} \]
\[ z(t) = [z_1(t), z_2(t), \ldots, z_n(t)]^T, \quad z(t-h) = [z_1(t-h_1), z_2(t-h_2), \ldots, z_n(t-h_n)]^T. \]

For further discussion, we need to introduce some lemmas:

**Lemma 1.** (Schur complement) Given constant symmetric matrices \( \Sigma_1, \Sigma_2, \Sigma_3 \) where \( \Sigma_1 = \Sigma_1^T \) and \( 0 < \Sigma_2 = \Sigma_2^T \), then \( \Sigma_1 + \Sigma_2^T \Sigma_3^{-1} \Sigma_2 < 0 \) if and only if \( \begin{pmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{pmatrix} < 0 \) or \( \begin{pmatrix} -\Sigma_2 & \Sigma_3^T \\ \Sigma_3 & \Sigma_2 \end{pmatrix} < 0 \).

**Lemma 2.** For any constant symmetric positive-definite matrix \( W \), a positive scalar \( \sigma \) and a vector function \( f(t) : [0, \sigma] \to \mathbb{R}^n \) such that the integrations in the following are well defined, then
\[ \sigma \int_0^\sigma f'(t)Wf(t)dt \geq \left( \int_0^\sigma f(t)dt \right)^T W \left( \int_0^\sigma f(t)dt \right). \]
For further analysis, define a new vector integral operator \( D(z(t)) = [D(z_1(t)), D(z_2(t)), \ldots, D(z_n(t))]^T \) as follows:
\[ D(z(t)) = z(t) + \bar{A}_\xi \int_{t-h}^t z(s)ds - \bar{A}_\xi z(t-h) \tag{5} \]

Then, we can get the following property of this operator as

**Theorem 1.** For operator \( D(z(t)) : \phi_0 \to \mathbb{R}^n \), we have
\[ \frac{dD(z(t))}{dt} = (\bar{A}_\xi + \bar{A}_\xi)z(t). \tag{6} \]

**Proof.** Let \( \bar{A}_\xi = \bar{A}_\xi^{(t)}_{(\text{new})}, \) \( k = 0, 1, 2. \) Since
\[ D(z(t)) = \begin{pmatrix} z_1(t) \\ z_{11}(t) \\ \vdots \\ z_{1n}(t) \\ z_{21}(t) \\ \vdots \\ z_{2n}(t) \\ \vdots \\ z_{n1}(t) \\ \vdots \\ z_{nn}(t) \end{pmatrix} \begin{pmatrix} \int_{t-h}^t z_{1}(s)ds \\ \int_{t-h}^t z_{11}(s)ds \\ \vdots \\ \int_{t-h}^t z_{1n}(s)ds \\ \int_{t-h}^t z_{21}(s)ds \\ \vdots \\ \int_{t-h}^t z_{2n}(s)ds \\ \vdots \\ \int_{t-h}^t z_{n1}(s)ds \\ \vdots \\ \int_{t-h}^t z_{nn}(s)ds \end{pmatrix} \]
and
\[ \frac{dz(t)}{dt} = \begin{pmatrix} z_1(t) \\ z_{11}(t) \\ \vdots \\ z_{1n}(t) \\ z_{21}(t) \\ \vdots \\ z_{2n}(t) \\ \vdots \\ z_{n1}(t) \\ \vdots \\ z_{nn}(t) \end{pmatrix} \begin{pmatrix} \int_{t-h}^t z_{1}(s)ds \\ \int_{t-h}^t z_{11}(s)ds \\ \vdots \\ \int_{t-h}^t z_{1n}(s)ds \\ \int_{t-h}^t z_{21}(s)ds \\ \vdots \\ \int_{t-h}^t z_{2n}(s)ds \\ \vdots \\ \int_{t-h}^t z_{n1}(s)ds \\ \vdots \\ \int_{t-h}^t z_{nn}(s)ds \end{pmatrix} \]

Along the trajectories of the system (4), the derivative of \( D(z_1(t)) \) is
\[ \frac{dD[z_1(t)]}{dt} = \int_{t-h}^t \frac{d\bar{A}_\xi^{(t)}}{dt} z_1(t)z_1(t-h) + \int_{0}^{\sigma} \bar{A}_\xi^{(t)} \frac{dz_1(t)}{dt} \frac{dz_1(t-h)}{dt} - \int_{0}^{\sigma} \bar{A}_\xi^{(t)} \frac{dz_1(t-h)}{dt} \frac{dz_1(t)}{dt} \]
\[ = \sum_{j=1}^{n} (\bar{A}_\xi^{(t)} + \bar{A}_\xi^{(t)}) z_1(t), \quad i = 1, 2, \ldots, n. \]
Namely \( \frac{d\mathbf{D}[z(t)]}{dt} = (\tilde{A}_h + \tilde{A}_i)z(t) \), which complete the proof.

**Remark 1.** By neutral stability theory, we can easily get that stability of operator \( \mathbf{D}(z(t)) \) given in (5) is \( \rho(\|E\|) < 1 \).

### 3. Controller Design

In this section, we shall establish a criterion for the existence and asymptotic stability of the controller (3) using the new integral operator defined by (5). The criterion will be expressed in terms of LMIs, which can be checked numerically using the effective LMI toolbox in MATLAB.

**Theorem 2.** For given positive constants \( h_i \ (i=1, 2, \ldots, n) \), suppose that \( \rho(\|E\|) < 1 \), then there exists an asymptotically stable dynamic output feedback controller (3) for system (1), if there exist positive scalars \( \varepsilon, \gamma \) positive-definite matrices \( S, Y, X \), and matrices \( \hat{A}, \hat{B}, \hat{C} \) satisfying the following matrix inequalities

\[
\Omega_1 Y \Omega_2 \Omega_3 - \hat{A}^T - \Omega_3 \hat{A}^T \leq 0,
\]

\[
\begin{bmatrix}
\Omega_1 & Y & \Omega_2 & \Omega_3 & -\hat{A}^T & -\Omega_3 & \hat{A}^T \\
\ast & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
\ast & \ast & \Omega_4 & \Omega_5 & -\hat{A}^T & -\Omega_3 & \hat{A}^T \\
\ast & \ast & -\gamma X & -\gamma Y & 0 & 0 & 0 \\
\ast & \ast & \ast & -\gamma I & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & -\varepsilon X (h^T)^T & -\varepsilon Y (h^T)^T \\
\ast & \ast & \ast & \ast & \ast & \ast & -\varepsilon I (h^T)^T \\
\end{bmatrix}
\]

\( I - \varepsilon \hat{A}_i \hat{A}_2 > 0 \), \( 1 - \varepsilon \hat{A}_i \hat{A}_2 > 0 \), \( \begin{bmatrix} Y & I \\ I & S \end{bmatrix} > 0 \)

where

\[
\Omega_1 = AY + YA^T + \hat{B}\hat{C} + \hat{C}^T\hat{B}^T + 2X
\]

\[
\begin{align*}
B&B_0 + B_1, & \Omega_2 = A + \hat{A}^T + 2Y \\
\Omega_3 = -YA^T - \hat{C}^T\hat{B}^T, & \Omega_5 = -(A^T S + C^T \hat{B}^T) \\
\hat{A} &= SAY + S\hat{B}\hat{C} + \hat{B}\hat{C}Y + N^T \hat{A} \hat{M}^T \\
\hat{A}_2 &= S^T A + A^T S + \hat{B}\hat{C} + C^T \hat{B}^T + 2I.
\end{align*}
\]

**Proof.** Consider the following legitimate Lyapunov functional

\[
V = \mathbf{D}(z(t))^T \mathbf{P} \mathbf{D}(z(t)) + \sum_{i=1}^{n} \int_{t-h_i}^{t} z_i^2(s)ds + \sum_{i=1}^{n} \int_{t-h_i}^{t} (s-t+h_i)z_i^2(s)ds
\]

where \( \mathbf{P} > 0 \). Taking the time derivative of \( V \) along the solution of system (6), we obtain

\[
\frac{dV}{dt} = 2 \mathbf{D}^T(z(t)) \mathbf{P} \mathbf{D}(z(t)) + \sum_{i=1}^{n} \int_{t-h_i}^{t} z_i^2(s)ds
\]

\[
- \sum_{i=1}^{n} \int_{t-h_i}^{t} z_i^2(s)ds - \int_{t-h_i}^{t} \varepsilon z_i^2(s)ds
\]

\[
= 2z^T(t)(\tilde{A}_h + \tilde{A}_i)^T \mathbf{P}[z(t) + \tilde{A}_i \int_{t-h}^{t} z(s)ds - \tilde{A}_h \mathbf{z}(t-h)]
\]

\[
+ 2z^T(t)(\tilde{A}_h + \tilde{A}_i)^T \mathbf{P}[\mathbf{z}(t) + \tilde{A}_i \int_{t-h}^{t} z(s)ds - \tilde{A}_h \mathbf{z}(t-h)]
\]

\[
\times \mathbf{P} \hat{A}_i \int_{t-h}^{t} z(s)ds + 2z^T(t)(\tilde{A}_h + \tilde{A}_i)^T \mathbf{P} \hat{A}_i \mathbf{z}(t-h)
\]

\[
+ 2z^T(t)(\mathbf{z}(t) - \mathbf{z}(t-h)\mathbf{z}(t-h) - \sum_{i=1}^{n} \int_{t-h}^{t} z_i^2(s)ds.
\]

Utilizing lemma 2, we have

\[
- \int_{t-h}^{t} z_i^2(s)ds \leq - \int_{t-h}^{t} z_i^2(s)ds - \int_{t-h}^{t} z_i^2(s)ds + \int_{t-h}^{t} z_i^2(s)ds
\]

Thus we get

\[
\frac{dV}{dt} \leq z^T(t)[2(\tilde{A}_h + \tilde{A}_i)^T \mathbf{P} + 2I]z(t)
\]

\[
+ 2z^T(t)(\tilde{A}_h + \tilde{A}_i)^T \mathbf{P} \hat{A}_i \int_{t-h}^{t} z(s)ds
\]

\[
- 2z^T(t)(\tilde{A}_h + \tilde{A}_i)^T \mathbf{P} \hat{A}_i \mathbf{z}(t-h)
\]

\[
- z^T(t-h)\mathbf{z}(t-h) - \frac{1}{(h^T)^2} \left( \int_{t-h}^{t} z_i^2(s)ds \right)^2
\]

\[
\leq z^T(t)[2(\tilde{A}_h + \tilde{A}_i)^T \mathbf{P} + 2I]z(t)
\]

\[
+ 2z^T(t)(\tilde{A}_h + \tilde{A}_i)^T \mathbf{P} \hat{A}_i \int_{t-h}^{t} z(s)ds
\]

\[
- 2z^T(t)(\tilde{A}_h + \tilde{A}_i)^T \mathbf{P} \hat{A}_i \mathbf{z}(t-h)
\]

\[
- z^T(t-h)\mathbf{z}(t-h) - \frac{1}{(h^T)^2} \left( \int_{t-h}^{t} z_i^2(s)ds \right)^2
\]

\[
\leq z^T(t)[2(\tilde{A}_h + \tilde{A}_i)^T \mathbf{P} + 2I]z(t)
\]

where \( \mathbf{P} > 0 \). Taking the time derivative of \( V \) along the solution of system (6), we obtain

\[
\frac{dV}{dt} = 2z^T(t)(\tilde{A}_h + \tilde{A}_i)^T \mathbf{P}[z(t) + \tilde{A}_i \int_{t-h}^{t} z(s)ds - \tilde{A}_h \mathbf{z}(t-h)]
\]

\[
+ 2z^T(t)(\tilde{A}_h + \tilde{A}_i)^T \mathbf{P}[\mathbf{z}(t) + \tilde{A}_i \int_{t-h}^{t} z(s)ds - \tilde{A}_h \mathbf{z}(t-h)]
\]

\[
\times \mathbf{P} \hat{A}_i \int_{t-h}^{t} z(s)ds + 2z^T(t)(\mathbf{z}(t) - \mathbf{z}(t-h)\mathbf{z}(t-h) - \sum_{i=1}^{n} \int_{t-h}^{t} z_i^2(s)ds.
\]
\[
\Sigma' = \begin{pmatrix}
(1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\
(2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\
(3,3) & \gamma Y & 0 & 0 & 0 & 0 \\
(5,5) & -\frac{\varepsilon Y}{(h')^2} & 0 & 0 & 0 & 0 \\
& & & & & < 0
\end{pmatrix}
\]

where

\[
\begin{align*}
(1,1) &= \Omega_1 + 2(YY^T + X), \quad (1,2) = \Omega_2 \\
(1,3) &= \Omega_1, \quad (1,4) = -\hat{A}^T, \quad (1,5) = -\Omega_3 \\
(1,6) &= \hat{A}^T, \quad (2,2) = \Omega_1, \quad (2,3) = -A^T \\
(2,4) &= \Omega_1, \quad (2,5) = A^T, \quad (2,6) = -\Omega_5 \\
(3,3) &= -\gamma(YY + X), \quad (5,5) = -\frac{\varepsilon}{(h')^2}(YY + X)
\end{align*}
\]

By Lemma 1, (12) is equivalent to the first inequality in Theorem 2. Also, the operator \(D(z(t))\) is stable if the condition \(\rho(||E||) < 1\) holds. From stable theory of neutral system, we conclude that system (1) is asymptotically stable, which complete the proof.

Remark 2. Given any solution of the matrix inequalities in Theorem 2, a corresponding controller of the form will be constructed as follows:

1. Using the solution \(X\) to compute the invertible matrices \(M\), which satisfies the relation \(X = MM^T\).
2. Using the matrix \(M\) to compute the invertible matrix \(N\), which satisfies \(MN^T = I - YS\).
3. Utilizing the matrices \(M\) and \(N\) obtained above to solve for \(B, C, A\) (in this order).
4. Checking the inequalities \(I - \gamma \hat{A}_b^T \hat{A}_b > 0\) and \(I - \varepsilon \hat{A}_b^T \hat{A}_b > 0\) satisfied or not.

To illustrate the design procedure, we shall give a numerical example provided later.

Remark 3. When \(h_1 = h_2 = \cdots = h_n = h\), system (1) becomes a single-time delayed neutral differential system, hence, Ref. [15] is a special case.
4. Numerical Example

In this section, we shall employ one numerical example to demonstrate the correctness and feasibility of the obtained result.

Example 1. Consider the following neutral differential system with two delays

\[
\begin{align*}
\frac{dx(t)}{dt} &= Ax(t) + E \frac{dx(t-h)}{dt} + B_0 u(t) + B_1 u(t-h) \\
y(t) &= Cx(t)
\end{align*}
\]  
(14)

where

\[
A = \begin{pmatrix}
-0.02 & 0 \\
0.1 & -0.01
\end{pmatrix}, \quad E = \begin{pmatrix}
0.2 & 0 \\
0 & 0.2
\end{pmatrix}
\]

\[
B_0 = \begin{pmatrix}
-0.3 & 0 \\
-0.01 & -0.3
\end{pmatrix}, \quad C = \begin{pmatrix}
0.01 & 0 \\
0.01 & -0.02
\end{pmatrix}
\]

\[
B_1 = \begin{pmatrix}
0.1 & 0 \\
0.01 & 0.1
\end{pmatrix}, \quad h_1 = 0.1, h_2 = 0.2.
\]

Here, we want to design a dynamic output feedback controller for system (14), which guarantees the asymptotic stability of the closed-loop system. First, it is easy for us to check that the operator \(D(z(t))\) given in Theorem 1 is stable, since \(\rho(\|E\|) < 1\). Next, by applying the Theorem 2 to system (14) and checking the feasibility of LMIs conditions in Theorem 2, if set \(\gamma = 5.1, \epsilon = 10.2\), we can find that the LMIs are feasible and we can obtain the solutions of the inequalities as follows

\[
S = \begin{pmatrix}
10.4942 & 0.4048 \\
0.4048 & 9.6929
\end{pmatrix}, \quad X = \begin{pmatrix}
2.3213 & -0.0088 \\
-0.0088 & 2.3373
\end{pmatrix}
\]

\[
Y = \begin{pmatrix}
0.3681 & 0.0024 \\
0.0024 & 0.0401
\end{pmatrix}, \quad \hat{A} = \begin{pmatrix}
-0.9895 & 0.1610 \\
-0.1599 & -1.0052
\end{pmatrix}
\]

\[
\hat{B} = \begin{pmatrix}
313.4652 & 34.9027 \\
53.1646 & 126.2187
\end{pmatrix}, \quad \hat{C} = \begin{pmatrix}
31.1136 & -0.1046 \\
1.4791 & 31.2974
\end{pmatrix}
\]

\[
M = \begin{pmatrix}
1.5236 & -0.0029 \\
-0.0029 & 1.5288
\end{pmatrix}, \quad N = \begin{pmatrix}
-1.8802 & -0.1264 \\
-0.1166 & -1.8948
\end{pmatrix}
\]

and the corresponding positive matrix \(P\) is

\[
P = \begin{pmatrix}
10.4942 & 0.4080 & -1.8802 & -0.1264 \\
0.4080 & 9.6929 & -0.1166 & -1.8948 \\
-1.8802 & -0.1166 & 1.5236 & -0.0029 \\
-0.1264 & -1.8948 & -0.0029 & 1.5288
\end{pmatrix}
\]

Then, we can get

\[
A_c = \begin{pmatrix}
-22.8859 & 0.2524 \\
0.8167 & -20.9759
\end{pmatrix}
\]

Finally, we can easily check that the conditions \(1 - \gamma \hat{A}^* \hat{A} > 0\) and \(1 - \epsilon \hat{A}^* \hat{A} > 0\) are satisfied in light of \(h^* = 0.2\), thus \(A_c, B_c, C_c\) are the stabilizing dynamic output feedback controller for system (14).

From Fig. 1, we can see that the concerned system (14) is asymptotically stabilized by the dynamic control law (3).

5. Conclusions

This paper investigates the problem of stabilization for a class of neutral differential systems with multi-time delays in control input. By introducing a new integral operator, a new asymptotically stable criterion is obtained in terms of LMIs which can be easily solved by various efficient convex optimization algorithms. Based on the new established criterion, a dynamic output feedback controller which guarantees the asymptotic stability of the concerned system is also given. Numerical simulation shows that the new design method is valid. Our criterion generalizes some previous results obtained in the reference cited therein. On the other hand, we should like to point out that it is possible to generalize our main result to more complex dynamical systems. The corresponding results will appear in the future.

References


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