Features and Prototypes of HTS High $Q$ Resonant Circuit

Jian-Xun Jin

Abstract—High $T_c$ superconductor (HTS) technology has been used to develop a unique high $Q$ resonant circuit. Such circuit or device has some special characteristics such as very high voltage generation. Theoretical study and experimental approaches have proceeded for the concept verification. This paper presents the theory about this high $Q$ resonant circuit. The operation principle of the circuit is described. A practical prototype for HTS high voltage generation is also demonstrated. The experiment result shows that very high voltages can be achieved by the developed method using HTS technology.

Index Terms—Electronic resonant circuit, high $Q$ inductor, high $T_c$ superconductor, high voltage, power electronic switch.

1. Introduction

High $T_c$ superconductors (HTS) have been well developed in the last twenty years and become available for practical applications\(^1\)[1]\(^2\]. A HTS resistor-capacitor-inductor (RCL) series resonant circuit, controlled by a power electronic switch, has special features\(^3\). The advanced circuit using a HTS inductor has been studied with regard to its quality factor $Q$, and voltage or current generation. This resonant circuit mainly consists of an inductor, a capacitor, a DC battery source, and an electronic switch. As basic operational principle, a low voltage DC power source can be used, and its polarity is reversed at a certain frequency controlled with a power electronic switch. This circuit can be used to generate high voltage and current generation from a low voltage source by using a HTS inductor. And the generated high voltage is proportional to the circuit $Q$ value. The resistance in the circuit will limit the $Q$ value and therefore the output voltage that can be achieved in practice. However HTS technology can dramatically reduce the resistance and presents a very high $Q$ value circuit.

In this paper, the HTS high $Q$ resonant circuit is concerned. Section 2 presents the theoretical analysis of the circuit. The operation principle is described in Section 3 with a practical prototype of circuit implementation. Section 4 gives the discussions of some characteristics of the circuit.

2. Theory

2.1 $Q$ Value and Features of a Resonant Circuit

From circuit theory, RCL series resonant circuits are characterized by its quality factor $Q = \omega_0 L / R$, where $\omega_0 = [(LC)^{-1} - R^2 (2L)^{-2}]^{1/2}$ is the resonant frequency. The potential difference across the capacitor at resonance, i.e. the maximum voltage $V_{\text{Cmax}}$ across the capacitor, is $Q$ times as great as the applied emf $V_p$ (rms) to the circuit. For sinusoidal power supply, $V_{\text{Cmax}} = Q V_p$; for DC power supply with an electronic switch to reverse the polarity, $V_{\text{Cmax}}$ can be expressed as $V_{\text{Cmax}} = Q V_p'$, where $V_p'$ is the effective voltage of the low voltage power source. If the used switching controller has rectangular switching wave form with the low voltage source polarity switching frequency $f$, the build-up voltage wave form $F(t)$ can be expressed by Fourier series as

$$F(t) = (4V_p / \pi)[\sin cot + (1/3)\sin 3cot + (1/5)\sin 5cot + \cdots].$$

The generator only resonates on the first harmonic with amplitude of $4V_p/\pi$. Therefore, $V_p' = 4V_p / \pi$, and the maximum build-up voltage is $V_{\text{Cmax}} = Q(4V_p / \pi)$.

The resistance of the RCL resonant circuit can be reduced by introducing a superconducting inductor; the circuit $Q$ value then will be dramatically increased, therefore leading to a very high voltage across the capacitor.

2.2 Resistance-Less Circuit

For a RCL series resonant circuit, when a DC source is switched to the circuit, the instantaneous current $i(t)$ and capacitor voltage $v_c(t)$ are

$$i(t) = \exp\left(-\frac{Rt}{2L}\right)\left[\frac{(V_{C0}+V_p)\sin \omega t}{\omega L}\right]$$

$$v_c(t) = (V_{C0}+V_p)\left[1-\exp\left(-\frac{Rt}{2L}\right)(\cos \omega t + \frac{R}{2L \omega} \sin \omega t)\right] - V_{C0}$$

where $R < 2/(L/C)^{1/2}$, $V_{C0}$ is the initial capacitor voltage. Both equations describe decaying sinusoids, with $v_c(t)$ approaching a steady state value of $V_p$, and $i(t)$ approaching a steady state value of zero.

Now in a circuit using a HTS inductor and without separate resistor, then $R$ will become very small. If $R = 0 \Omega$,
equations (1) and (2) can be simplified to
\[ i(t) = \frac{(V_{C0} + V_B) \sin \omega t}{\omega L} \quad (3) \]
\[ V_C(t) = -(V_{C0} + V_B) \cos \omega t + V_B \quad (4) \]
where \( \omega_x = (LC)^{-1/2} \). Equations (3) and (4) describe constant magnitude sinusoids, with the average values of \( i(t) \) and \( V_C(t) \) being zero and \( V_B \), respectively.

When the DC source is applied, \( V_C(t) = V_C(0) = -V_{C0} \). Half a resonant cycle later, this voltage will be increased to
\[ V_C(t) = V_C(\pi/\omega) = -(V_{C0} + V_B)(-1) + V_B = V_{C0} + 2V_B. \quad (5) \]

At this time, if the DC source is disconnected and reconnected in the opposite polarity for the next half cycle, the initial capacitor voltage \( V_{C0} \) is changed to \( V_{C0,new} \) given by \( V_{C0,new} = -V_C(t) = -(V_{C0} + 2V_B) \). Half a cycle later, \( V_C(t) = V_C(\pi/\omega) \) becomes
\[ V_C(t) = -(V_{C0} + 2V_B) + (V_B)(1) + (V_B) = -(V_{C0} + 4V_B). \quad (6) \]
If the DC source polarity is reversed every half cycle thereafter, then
\[ V_C(t) = (V_{C0} + 6V_B); \quad -(V_{C0} + 8V_B); \quad (V_{C0} + 10V_B); \ldots. \quad (7) \]
This is the build-up voltage for an ideal non-resistive circuit. The positive and negative peak voltages can be described by (7). Consequently in a resistance-less circuit, the voltage across the capacitor \( C \) after \( n \) cycles will be
\[ V_C(n) = (-1)^{n+1} (V_{C0} + 2nV_B). \quad (8) \]

### 2.3 Practical Resistive Circuit
From (3), when \( t = \pi/\omega \) and \( i = 0 \), if the DC source polarity is reversed by the electronic switch bridge in Fig. 1, the capacitor voltage is given by
\[ V_{C1} = (V_{C0} + V_B)[1 + \exp(-R\pi/(2Lo))] - V_{C0}. \quad (9) \]
After the polarity is changed \( n \) times, the capacitor voltage becomes
\[ V_{Cn} = (V_{Cn-1} + V_B)[1 + \exp(-R\pi/(2Lo))] - V_{Cn-1}. \quad (10) \]
If \( V_{C0} = 0 \), we have
\[ V_{Cn} = V_B[1 + 2e^{-R\pi/(2Lo)} + 2e^{-2R\pi/(2Lo)} \ldots + 2e^{-(n-1)R\pi/(2Lo)} + e^{-nR\pi/(2Lo)}] \]
\[ = V_B(1 + e^{-nR\pi/(2Lo)}) + 2V_B \sum_{i=1}^{n-1} e^{-iR\pi/(2Lo)}. \quad (11) \]
Assuming that the power supply polarity changes at \( t = n\pi/\omega \), after the polarity is changed \( n \) times, the build-up \( V_{Cn} \) is
\[ V_{Cn} = V_B(1 + e^{-nR\pi/(2Lo)}) + 2V_B \frac{e^{-R\pi/(2Lo)}}{1 - e^{-R\pi/(2Lo)}}. \quad (12) \]

When \( n \to \infty \), then \( V_{Cn} \to V_{C_{max}} \), and
\[ V_{C_{max}} = \lim_{n \to \infty} V_{Cn} = V_B(1 + \frac{2}{e^{R\pi/(2Lo)}} - 1). \quad (13) \]

From (13), when \( R \to 0 \), then \( V_{C_{max}} \to \infty \); when \( R \to \infty \), then \( V_{C_{max}} \to V_B. \) Since \( Q = \alpha L/R \), equation (13) can be expressed as
\[ V_{C_{max}} = V_B + \frac{2V_B}{(e^{\pi/(2Q)} - 1)}. \quad (14) \]

To obtain the \( v_C(n) \) value has different methods. The equation describing the positive and negative voltages, for each DC source polarity reversal is
\[ v_C(n) = [a_nK^n + a_{n-1}K^{n-1} + \ldots + a_1K + a_0]V_B + [b_nK^n + b_{n-1}K^{n-1} + \ldots + b_1K + b_0]V_{C0} \quad (15) \]
where \( n \) is the iteration number, \( K = 1 + \exp[-R\pi/(2Lo)] \), and the values of \( a_i \) and \( b_i \) are shown in Table 1 for the first five iterations as example. The table can be expanded for higher iterations by inspection. A coefficient can be calculated by taking the coefficient value directly above and subtracting the value of the coefficient in the above right position, except for the \( i = 0, 1 \) and \( n \) positions which are obvious.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Coefficient values for ( a_i ) and ( b_i ) in (15)</th>
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<tbody>
<tr>
<td>( n )</td>
<td>( a_1 )</td>
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<tr>
<td>0</td>
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<td>1</td>
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<td>5</td>
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\[ \frac{dv_C(t)}{dt} \bigg|_{t=0} = -\alpha K_1 + \omega_0 K_2 = 0. \quad (17) \]

When \( K_1 = -V_n, K_2 = -V_n \omega_0 \Delta t \),
\[ v_c(t) = -V_b e^{-\alpha t} (\cos \omega_d t + \frac{\alpha}{\omega_d} \sin \omega_d t) + V_b \]

\[ = V_b \left[ 1 - (\omega_b/\omega_d) e^{-\alpha t} \cos(\omega_d t - \theta) \right] \quad t \geq 0 \quad (18) \]

where \( \theta = \arctan(\alpha / \omega_d) \).

3. Operation Principle

Fig. 1 shows a resonant circuit controlled by a power electronic switch. It consists of an electronic bridge which applies and periodically reverses a low voltage DC source polarity to the circuit, e.g. a bridge of four electronic switches such as silicon controlled rectifiers (SCRs) used in this case.

This circuit is accomplished by a control circuit triggering the alternate pairs of SCRs at a selected rate. Practically, the timing comes from a 4017 decade counter clocked by an adjustable oscillator. The 4017’s Q0 and Q4 outputs alternately enable logic gates to switch a burst of high frequency pulses to the primary winding of one of a pair of SCR’s trigger transformers. High voltage isolation is provided by these transformers. The AC energy on the secondary winding is rectified to provide reliable triggering of the SCR. The Q8 output of the 4017 is used to reset the 4017 so that a symmetrical triggering cycle results. Fig. 2 shows the timing sequence for the thyristor triggering. Fig. 3 shows a graph illustrating the voltage and current build-up as the principle of operation, where voltage plateaux exist between charge currents, i.e. minimum plateau time for relay operation. The practically designed device circuit frame is shown in Fig. 4.

Suppose that the initial state of the bridge circuit in the Fig. 1 is: SCRs off, capacitor discharged; no current flowing, S1 and S4 are both triggered, battery voltage \( V_b \) is applied to the series resonant circuit consisted of \( L \) and \( C \). If the trigger pulse is maintained until the SCR latching current is reached and any loss is not considered, a current will rise sinusoidally to a maximum and drop to zero whereupon the SCRs will cease conducting due to load commutation. The length of this charge pulse is one half of the natural resonant period of the \( CL \) circuit. The voltage left on the capacitor will be twice the battery voltage \( V_b \). Now if at a later time S2 and S3 are triggered, the battery voltage will be placed in series with the voltage left on capacitor \( C \) and this cycle will again add twice the battery voltage to \( C \) but with opposite sign, thus: \( 2V_b, -4V_b, 6V_b, -8V_b, 10V_b, \ldots \), will be the sequence of voltages produced across the capacitor. Consequently, repeated cycles will raise the absolute voltage of \( C \) until losses in the resonant circuit cause a voltage plateau to be reached in a practical resistive circuit. Now if the period between SCR switching is enlarged, the slow voltage loss in \( C \) between
charge sinusoids will cause the voltage left on $C$ to be stabilized at lower levels. This gives some means of voltage control, although if the $Q$ is changed while the system is running, the voltages and currents will change as a result.

Fig. 5. The first prototype device for HTS high voltage generation experiment.

Fig. 6. Experimental voltage built up with lower inductor.

An experimental prototype has been developed for verification of the special voltage generation method. For the first prototype device, four silicon controlled rectifiers (SCRs) are connected as shown in Fig. 5. The device using 12 V DC batteries has 1.2 kV rated output voltage and allows 20 A current. Fig. 6 demonstrates its practical testing result using $L = 0.55$ mH, $C = 2.2 \ \mu F$, and only with the switch internal resistance. Fig. 7 shows a higher voltage built up by using $C = 2.2 \ \mu F$ and $L = 188$ mH with 4.74 $\Omega$ inductor resistance and the same switch internal resistance.

Fig. 7. Experimental voltage built up with higher inductor.

Another modified experimental prototype device, as shown in Fig. 8, developed in the University of Electronic Science and Technology of China (UESTC), uses MOSFET switches and a 12 V DC source. This device has frequency range of 10 Hz-1 kHz, maximum output voltage 1.8 kV, and maximum operation current 10 A.

Fig. 8. The modified prototype of resonant controller developed by UESTC.

4. Discussions

4.1 Circuit Features for Voltage Generation

With a HTS, the resonant circuit can be developed to be a method of generating high voltage from a low voltage source. This method can achieve high voltages by using a high $Q$ circuit and with an appropriate choice of circuit components. As an example, Fig. 9 shows the envelope of $v_C(t)$ in condition of $R = 0.05 \ \Omega$, $C = 20 \ \mu F$ and $L = 20$ mH. By limiting the number of iterations, or by employing some form of voltage sensing, it is possible to generate a pre-determined set voltage. Based the second prototype device mentioned above, the output voltage has the highest value at resonant point, as shown the Fig. 10, where the circuit is implemented with $L = 331$ mH, $C = 220$ nF and $R = 18.48 \ \Omega$.

Fig. 9. Envelope of $v_C(t)$ for a sample circuit.

Fig. 10. Output with frequency scan.

Practically, any circuit resistance causes energy
dissipation, and for each reversal of the DC source polarity the corresponding increase in voltage is less than $2V_{B}$, and the increase of the voltage magnitude gets smaller along with iteration increase. Any resistance in the circuit will limit the final achievable voltage. The circuit resistance is mainly caused by the inductor as well as the DC source, electronic switches, wires, and connections. In practice the final voltage across the capacitor would reach a finite maximum value. This would occur as a result of the small voltage increase per cycle being balanced by the same magnitude voltage loss per cycle due to leakage effects.

### 4.2 Circuit Current

To reduce the circuit resistance to near zero leads to an infinite circuit quality factor $Q$ at resonance. In a series resonant circuit with a power supply $V_p = V_m \sin \omega t$, the rms current $I_{max}$ in the circuit at resonance is $I_{max} = V_m / (R \sqrt{2})$, and the rms voltage across the capacitor $V_{Cmax}$ is $V_{Cmax} = X_C I_{max} = (\omega_0 C)^{-1} [V_m / (R \sqrt{2})]$. By assuming that the circuit resistance $R$ is zero, the circuit then has infinite $Q$ at resonant frequency $\omega_0 = (LC)^{-1/2}$. This leads to infinite values of $v_C$ and $i$ generated. This device therefore is able to provide controls of both high voltages and high currents. Fig. 11 shows the circuit current changes at different switching frequencies in the same circuit as the Fig. 10 but $R$ becomes 0.33 $\Omega$. Simulation results for a sample circuit are also presented in Fig. 12 and Fig. 13, where $i$ represents the circuit current.

![Fig. 11. Circuit current with frequency scan.](image1)

![Fig. 12. Circuit current with $R = 3 \Omega$, $C = 20 \mu F$, and $L = 20 mH$.](image2)

![Fig. 13. Envelope current with $R = 0.05 \Omega$, $C = 20 \mu F$, and $L = 20 mH$.](image3)

### 4.3 HTS High Q Inductor

HTS technology is critical for this method to be practical. HTS wires can be used to achieve the very high $Q$ inductor to make this method viable; on the other hand the conventional inductor technique can not make this method applicable. The high $Q$ inductor is able to be made by using Ag-clad (Bi,Pb)$_2$Sr$_2$Ca$_2$Cu$_3$O$_{10+x}$ HTS wires, which have potential capability to make the inductor winding. Fig. 14 shows an experimental HTS inductor made for the technology verification, where its magnetic field characteristic has suitable performance as the obtained analysis result shown in Fig. 15. Its main parameters are $h70 \text{mm}$/$\phi64 \text{mm}$/$364 \text{V}$/$20 \text{A}$/ $J_c=8 \times 10^6 \text{A/m}^2$/$~0.1 \text{T}$. This HTS wire has (i) high engineering critical current density $J_c > 10^4 \text{A/cm}^2$ for 77 K operation, this wire is readily to handle heavy currents as potentially required, (ii) high magnetic field tolerance when lower the operational temperature, (iii) mechanical flexibility, and (iv) long length. It can be employed for design of the HTS inductor to suit this application$^{[4]}$-$^{[6]}$.

The HTS inductor virtually has no resistance for a DC current operation; however loss will be generated even at a relative low value for low frequency AC application, which however does not affect this application significantly$^{[7]}$. The circuit current $i$, which can be calculated from $v_C$, is required for the design of the HTS inductor, and the circuit $di/dt$ is also required for the HTS inductor design as well as the design of the electronic switch.
4.4 Infinite Voltage Generation

Fig. 16 shows the voltage outputs at different frequencies in the same circuit as used for the Fig. 10 but $R$ becomes 0.33 $\Omega$. The resonant output voltage can be very high as the circuit $Q$ becomes high.

![Fig. 16 High voltage at resonant frequency.](image)

Due to the insulation of the HTS inductor, and also of the circuit components, the practical output voltage has to be limited. In order to achieve infinite output voltage practically, a method has been developed using voltage superimpose techniques, consequently any required high voltages can be achieved\[8\].

4.5 Applicable Features

This HTS resonant method can provide a very high voltage at a low load current for universal high voltage applications. When the load resistance across the capacitor is low, the maximum build up voltage is considerably reduced. Consequently the device is only capable of supplying a high resistance or high impedance load. The possible applications in electrical engineering include partial discharge testing and pressure testing of electrical insulation systems, etc., which do not require low impedance loads to be driven. This method can also be readily developed to generate or to control a very large potential current to an inductive circuit e.g. charging a superconducting magnet. Therefore this method analyzed is able to provide generation and control of both high voltages and high currents.

5. Conclusions

A resonant circuit with a power electronic controller has been verified theoretically and practically. The method developed enables voltages to be increased rapidly to a value many times greater than the input low voltage source. A potential very high voltage can be achieved by using a very high $Q$ inductor, which can be made by using HTS technology. The device built with a HTS can generate very high voltages for any high impedance load applications, to use for HTS AC loss study, and also for high current controls in any inductive circuit.

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References


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