Analysis of Resolution of Bistatic SAR

Tian-Ge Shao and Jian-Guo Wang

Abstract—In this paper, the spatial resolutions at different directions of bistatic synthetic aperture radar (BiSAR) have been derived from the ambiguity function. Compared with monostatic signal to noise ratio, BiSAR’s resolutions of a fixed point target are varying with slow time since BiSAR system is space-variant. Constraints for the assumption of space-invariant bistatic topology are proposed in the paper. Moreover, under the assumption of invariance, the change of resolutions at different points in the image scene is taken into account, and we have specified two key parameters that affect resolutions directly and analyzed the way how they influence on the resolutions.

Index Terms—Ambiguity function, bistatic SAR, resolution.

1. Introduction

The researches of bistatic synthetic aperture radar (BISAR) in last decade have attracted more and more attentions over the world, due to its predominant advantages to monostatic one. Spatial resolution, as one of the crucial parameters of bistatic signal to noise ration system, is discussed in [1]-[4]. In [5], the resolution of BiSAR is described with a simple geometry, which is only applicable for the stationary or parallel mode. In [6], a method on the use of gradient is brought out and can be applied to the general configurations. In [7] and [8], radar function and ambiguity function for BiSAR has been discussed. Opposed to monostatic one, BiSAR configuration is spatially shifted and its resolution is correspondingly space-shifted and time-shifted. In this paper, spatial resolutions are deduced from ambiguity function and a comprehensive insight into the problem of resolutions is obtained through deep researches on the most influential factors of resolution. Firstly, the resolutions at different directions are derived from ambiguity function. Then the constraints for assumption of invariant system are put forward and region of required resolution in the image scene is demonstrated to totally depend on BiSAR topology. Finally, the paper has specified the key parameters that affect the resolution directly and the way in which they influence on the resolution.

2. Bistatic SAR Geometry

The geometrical situation of BiSAR is sketched in Fig. 1. In rectangular coordinate $xyz$, $V_T$ and $V_R$ represent the velocity vector of the transmitter and receiver respectively, $U_{T0}$ and $U_{R0}$ are the position vectors of transmitter and receiver, respectively, pointing to the center of the aperture. $P$ represents the position of the target point of interest.

$\begin{align*}
V_T &= U_{T0} + uV_T \\
V_R &= U_{R0} + uV_R
\end{align*}$

According to the stop-and-go approximation, the point target response can be described by slow time $u$ and fast time $t$. Similarly, for the velocities of transmitters and receivers are far smaller than the electronic propagation speed, $U_T$ and $U_R$, which is the instantaneous position of transmitter and receiver with respect to slow time, can be represented as follows:

$\begin{align*}
U_T(u) &= U_{T0}(u) + uV_T \\
U_R(u) &= U_{R0}(u) + uV_R
\end{align*}$

Let the transmitted signal be $s(t) = h(t)\exp(j2\pi ft)$, $f$ be carrier frequency, $h(t)$ be the complex envelope and the received signal from the point $P$ can be expressed as:

$$f_p(u, t) = h(t - \tau_p)\exp(-j2\pi\tau_p).$$

where

$$\tau_p(u) = \frac{1}{c}||P - U_T(u)|| + ||P - U_R(u)||.$$

Doppler frequency without expansion is represented as:

$$f = \frac{1}{\lambda} \left[ V_T \cdot \frac{U_{T0} - P}{||U_{T0} - P||} + V_R \cdot \frac{U_{R0} - P}{||U_{R0} - P||} \right].$$

3. Bistatic Ambiguity Function

According to the definition of ambiguity function in [3], we can get expression of ambiguity function:
\( \chi(P, P') = \frac{\int f_p(t,u) f'_p(t,u) dt du}{\sqrt{\int [f_p(t,u)]^2 dt du} \int [f'_p(t,u)]^2 dt du} \)

\[ = \frac{\int h(t - \tau_p(u)) h'(t - \tau_p(u)) \exp[j 2 \pi f_e(t - \tau_p(u))] dt du}{\sqrt{\int [h(t - \tau_p(u))]^2 dt du} \int [h(t - \tau_p(u))]^2 dt du} \]  

(3)

where \( P' \) is an arbitrary point near the point \( P \). \( f_p(t,u) \) and \( f'_p(t,u) \) are the received signals from \( P \) and \( P' \), respectively. Equation (3) shows that the ambiguity function is the correlation coefficient of the response of two point targets.

In frequency domain, ambiguity function can be described as:

\[ \chi(P, P') = \frac{\int P(f) \exp[j 2 \pi f (\tau_p u - \tau_p u)] df}{\sqrt{\int [P(f)]^2 df}} \times \int \exp[j 2 \pi f_e (\tau_p u - \tau_p u)] du \]  

(4)

where \( P(f) \) is the power spectrum of the transmitted signal. The first integral in (4) is caused by the complex envelope of transmitted signal, while the latter integral is caused by the carrier frequency. To simplify the ambiguity function, several assumptions can be made as follows on which all the first-order Taylor expansion in the following text are based on:

\[ |P| << |U_{10}|, |P| << |U_{90}|, L_g << |U_{90}| \text{ and } L_r << |U_{10}|. \]

\( L_g \) and \( L_r \) are the length of transmitting and receiving antenna apertures, respectively.

For convenience of expression, several definitions are made:

1. \( \tau_d \) is the time delay difference between two target points at \( u = 0 \).

\[ \tau_d = \frac{1}{c} \left[ |U_{10} - P| + |U_{90} - P| - |U_{10} - P'| - |U_{90} - P'| \right]. \]  

(5)

2. \( f_d \) is the Doppler frequency difference between two different target points at \( u = 0 \).

\[ f_d = \frac{1}{c} \left[ V_T \left[ \frac{U_{10} - P'}{U_{10} - P} \right] + V_R \left[ \frac{U_{90} - P'}{U_{90} - P} \right] - V_T \left[ \frac{U_{10} - P}{U_{10} - P} \right] - V_R \left[ \frac{U_{90} - P}{U_{90} - P} \right] \right]. \]  

(6)

Due to the above assumptions that length of synthetic aperture is much smaller than the distance from target to airplanes, approximation can be made as follow by using first-order Taylor expansion:

\[ 2 \pi f_e (\tau_p u - \tau_p u) \approx 2 \pi f_e \tau_d + 2 \pi f_d u. \]  

(7)

Substituting (7) into (4), the second integral in (4) can be written as

\[ \int \exp[j 2 \pi f_e (\tau_p u - \tau_p u)] du = \exp[j 2 \pi f_e \tau_d] \int \exp[2 \pi f_d u] du. \]  

(8)

\( g(f_d) \) is used to note the integral in (8), then

\[ g(f_d) = \int \exp[2 \pi f_d u] du = \sin(\pi L_{\text{eff}} f_d). \]  

(9)

Suppose \((-L_{\text{eff}}/2|V_b|, L_{\text{eff}}/2|V_b|)\) to be the range of integrating and \( L_{\text{eff}} \) be the effective coherent integration time, determined by bistatic configuration, including the aperture length and velocities of both transmitter and receiver. Different situations lead to different \( L_{\text{eff}} \) which are complex and discussed as a separate problem in [9].

Assume that transmitted signal is a narrow band signal and the frequency spectrum is narrow enough for \( P(f) \) to suppose the first integral in (4) can be approximated as:

\[ \int P(f) \exp[j 2 \pi f (\tau_p u - \tau_p u)] df = \int P(f) \exp[j 2 \pi f \tau_d] df. \]  

(10)

\( p(\tau_d) \) is used to note the integral in (10), then

\[ p(\tau_d) = \int P(f) \exp[j 2 \pi f \tau_d] df. \]  

(11)

Obviously, the first integral in (4) can be considered as the inverse Fourier transform of \( p(\tau_d) \). In other words, because \( P(f) \) is the frequency spectrum of transmitted signals, \( p(\tau_d) \) is the auto correlation of the complex envelope of the transmitted signal \( h(t) \).

From the above analysis, we can obtain the follow equation as simplified bistatic ambiguity function:

\[ \chi(P, P') = p(\tau_d) g(f_d) \exp[j 2 \pi f_d \tau_d]. \]  

(12)

The spatial resolution is mainly determined by the amplitude factor in (12).

### 4. Bistatic SAR Resolution

From (12), it is obviously that \( p(\tau_d) \) corresponds to time delay resolution while \( g(f_d) \) determines Doppler resolution. Let \( \delta_{\text{delay}} \) and \( \delta_{\text{doppler}} \) be \(-3\text{dB}\) widths of \( p(\tau_d) \) and \( g(f_d) \), respectively.

Using Taylor expansion of \( \tau_d \) with respect to \( P \) at \( P' = P \), we can obtain

\[ \tau_d \approx \frac{1}{c} \left( \frac{U_{10} - P}{U_{10} - P} + \frac{U_{90} - P}{U_{90} - P} \right) \cdot (P' - P). \]  

(13)

In the following, \( \Phi_p \) is the unit vector in the direction of \( U_{10} - P \), \( \Psi_p \) is the unit vector in the direction of \( U_{90} - P \), \( \beta \) represents the bistatic angle, and \( \Lambda \) is unit vector along the direction of bisector of bistatic angle.

\[ \Phi_p + \Psi_p = 2 \cos(\beta/2) \Lambda \]  

(14)

\[ \tau_d \approx \frac{1}{c} (\Phi_p + \Psi_p) \cdot (P' - P) = \frac{2}{c} \cos(\beta/2) \Lambda \cdot (P' - P). \]  

(15)

From (12) and (15), we can obtain that the optimum range resolution is along \( \Lambda \), and

\[ \delta_{\text{range}} = \frac{\delta_{\text{delay}} c}{2 \cos(\beta/2)} \Lambda. \]  

(16)
Similarly, using Taylor explanation of $f_d$ with respect to $P'$ at $P' = P$, we can obtain

$$f_d \approx \frac{1}{\lambda} \left( [I - (U_{T0} - P)(U_{T0} - P)^\tau]V_T \right) + \frac{1}{\lambda} \left( [I - (U_{R0} - P)(U_{R0} - P)^\tau]V_R \right)(P' - P). \quad (17)$$

Let $\omega_{TP}$ and $\Gamma$ be, respectively, the module and unit vector of the first factor in the folder, $\omega_{RP}$ and $\Theta$ be, respectively, the module and the unit vector of the second factor in the folder. Define $\varphi_\omega$ as the angle between $\Gamma$ and $\Theta$. In particular, $\varphi_\omega$ can represent the angle between transmitter’s and receiver’s velocity vectors when both the planes are boresight.

And let $\Xi$ and $\omega_{\text{SYN}}$ be respectively the module and unit vector of $\omega_{TP} \Gamma + \omega_{RP} \Theta$, there are

$$\omega_{TP} \Gamma + \omega_{RP} \Theta = \omega_{\text{SYN}} \Xi. \quad (18)$$

Substituting (18) into (17), we have

$$f_d \approx \frac{1}{\lambda} \left( \omega_{TP} \Gamma \omega_{TP} \Theta \right) \cdot (P' - P) = \frac{1}{\lambda} \omega_{\text{SYN}} \Xi \cdot (P' - P). \quad (19)$$

Using (12) and (19), the optimum azimuth resolution is along the direction of $\Xi$ and its complete expression can be described as follows:

$$\delta_{\text{azimuth}} = \delta_{\text{Doppler}} \frac{\lambda}{\omega_{\text{SYN}}} \Xi. \quad (20)$$

Usually, the resolutions in ground plane are what we concern about, so $\delta_{\text{range}}$ and $\delta_{\text{azimuth}}$ are projected into the plane ground and range resolution and azimuth resolution in ground plane can be obtained:

$$\delta_{\text{range, ground}} = \frac{\delta_{\text{range}}}{2\cos(\beta/2)} (1 - ZZ^\tau) \Lambda \quad (21)$$

$$\delta_{\text{azimuth, ground}} = \frac{\delta_{\text{Doppler}} \lambda}{\omega_{\text{SYN}}} (1 - ZZ^\tau) \Xi. \quad (22)$$

In (21) and (22), $Z$ represents the unit vector of $z$ in rectangular coordinate $xyz$. The range resolution and azimuth resolution are not perpendicular to each other any longer, so they can not completely express the resolvability of two point targets. Define $\theta$ to be the angle between the directions of $\Lambda(u)$ and $\Xi(u)$, then

$$\theta = \cos^{-1}(\Lambda(u) \cdot \Xi(u)) \quad (23)$$

and the cross range resolution can be defined as follows:

$$\delta_{\text{cross, range}} = \frac{|\delta_{\text{azimuth, ground}}|}{\sin(\theta)}. \quad (24)$$

The area of the resolution cell in plane ground can be obtained:

$$S_{\text{ground}} = \delta_{\text{range, ground}} \cdot \delta_{\text{azimuth, ground}}. \quad (25)$$

### 5. Performance of Resolution

Because BiSAR system is space-variant, in this section, the varying resolution of a fixed target point with respect to slow time $u$ will be studied first, then we will consider about the problem that how does the resolution of different target points change in the scene.

#### 5.1 Space-Invariance Restriction

Bistatic angle is the determinant parameter of the resolution, so we consider the cosine of bistatic angle of point P which can be expressed as

$$\cos \beta = \frac{|U_T(u) - P|^2 + |U_R(u) - P|^2 - |U_T(u) - U_R(u)|^2}{2|U_T(u) - P||U_R(u) - P|} \quad (26)$$

where $\beta$ is the bistatic angle of point P varying with slow time $u$. For convenience, suppose $P = 0$. The Taylor expansion of $\cos \beta$ as a function of $u$ at $u=0$ is listed below:

$$\cos \beta = \cos \beta_0 + k_u u \quad (27)$$

where $\beta_0$ is the initial $\beta$ at $u=0$.

$$k_u = \frac{D_0^T V_D}{U_{T0}||U_{R0}|} + \frac{|D_0|^2}{2|U_T||U_R|} \frac{\Phi_r \Psi_r(0)}{0} + \frac{V_T^T \Psi_r(0)}{0} \quad (28)$$

where $D_0 = U_{T0} - U_{R0}$, $V_T = V_T - V_R$. Using (26)-(28), the change in $\beta$ with a change in $u$ is given by

$$\Delta \beta = \frac{k_u}{\sqrt{1 - \cos^2 \beta_0}} \Delta u. \quad (29)$$

Bistatic angle and thus resolution change with slow time $u$ in bistatic space-invariance system. Equations (17)-(20) demonstrate that azimuth resolution, depending on $\beta$ bistatic space-invariant system. Equations (17)-(20)

$$\Delta \beta = \frac{k_u}{\sqrt{1 - \cos^2 \beta_0}} \Delta u. \quad (29)$$

If (30) can be satisfied, which means the change of range resolution induced by the variance of bistatic system is smaller than 1/8 of the origin value [10], we suppose that the bistatic system can be considered as invariant during time $\Delta t$. On the other hand, range resolution is comparatively more impressionable to change of $u$ than azimuth resolution, so we mainly use range resolution to determine the property of bistatic system:

$$\left| \frac{\delta_{\text{range}}(\Delta u)}{-\delta_{\text{range}}(0)} \right| < \frac{1}{8}; \quad (30)$$

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However, equation (31) is not analytically solvable, so we turn to numeric method. The relationship between the value of range resolution and slow time $u$ at different angles between $V_T$ and $V_R$ is shown in Fig. 2 and the main parameters are listed in Table 1.

<table>
<thead>
<tr>
<th>$B_T$ (MHz)</th>
<th>$V_T$ (m/s)</th>
<th>$V_R$ (m/s)</th>
<th>$U_{15}$ (km)</th>
<th>$U_{35}$ (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>200</td>
<td>200</td>
<td>35, 20, 6</td>
<td>17, 0, 6</td>
</tr>
</tbody>
</table>
From those curves in Fig. 2, it is obviously that the changes of range resolution with the slow time \( u \) in one coherent integration time under several different topologies can all perfectly tolerate the constraints in (30), including the most quick-changing curve. The result demonstrates the validation of constraints in (30) and thus (31). In addition, the simulations of point target image in the next segment 5.2 also justify (30).

Another observation can be achieved from Fig. 2. The varying rate of range resolution descends with the increase of the angle between \( V_{T} \) and \( V_{R} \). Take to the extremes, \( \beta \) changes very little with slow time \( u \) at the condition that transmitter and receiver have perpendicular trajectories while \( \beta \) drops the most quickly with \( u \) when the transmitter and receiver move in parallel trajectories. This result accords with the analysis in 5.2.

### 5.2 Point Target
Consider a special topology. Suppose that:
1) Both transmitter and receiver are boresight in strip mode.
2) Transmitter’s footprint is much more than that of receiver.
3) \( |V_{T}| = |V_{R}|, |U_{T0}| = |U_{R0}| \).

The expressions of resolutions (16) and (20) can be simplified as:

\[
|\delta_{\text{range}}| = \frac{c}{2B \cos(\beta/2)} \quad (32)
\]

\[
|\delta_{\text{azimuth}}| = \frac{\lambda}{\theta_{BW,R} \sqrt{1 + 2 \cos \varphi_{ao} + 1}} \quad (33)
\]

where \( \theta_{BW,R} \) is the transmitter’s antenna pointing angle. \( \varphi_{ao} \) is identical to the angle between \( V_{T} \) and \( V_{R} \). Obviously, bistatic angle \( \beta \) determines range resolution while \( \varphi_{ao} \) determines azimuth resolution. Fig. 3 shows the reconstruction image of point target at different \( \varphi_{ao} \).

The simulation parameters are listed in Table 1. In this experiment, the monostatic theoretical value of range and azimuth is 2 m and 1 m. When \( \varphi_{ao} = 0 \), the transmitter parallels the receiver, directions of range and azimuth resolution are still perpendicular, azimuth resolution is 2 m, double as monostatic one. With the increase of \( \varphi_{ao} \), the resolvability in azimuth decreases dramatically. The result of simulation in Fig. 3 is consistent with (32) and (33), which can well validate the above analysis of BiSAR resolutions.

### 5.3 Image Scene
Assume that the system can satisfy the restrictions of invariance, in another word, the system can be considered as a space invariant system. Take a scene of image into account. All the observations in this section are made at the observation time instant \( u = 0 \). Only the position of the target point \( P \) is considered as variable. Fig. 4 provides the information that how the resolution varies in different target point in the scene and point out the best area for imaging. Fig. 4(a) and Fig. 4(b), respectively, show the twisting lines of constant range resolution and azimuth resolution. Fig. 4(c) supplies a region of less-than-3 m resolution both in range and azimuth. Due to Fig. 4, it is easy to found that the two kinds of resolutions cannot achieve optimum in the same area in the scene. So in order to obtain better image, the initial topology
should be designed to satisfy the demand both in azimuth and range resolutions.

Fig. 4. Resolutions (a) Range resolution contours, (b) Azimuth resolution contours, (c) Region of less-than-3m resolution.

6. Conclusions

In this paper, we obtain the range and azimuth resolutions of bistatic SAR through the ambiguity function. Different from monostatic one, directions of range and azimuth are no longer perpendicular to each other. Furthermore, the resolutions are changing continuously with the slow time because BiSAR system is space-variant. Restrictions on which BiSAR system can be considered as invariant are given in Section 5.1 and are justified by simulations. An analysis of resolution varying with respect to the position in the image scene substantiates that range and azimuth resolution cannot achieve optimum at the same area in the scene and suitably designed topology is needed to satisfy the demand both in azimuth and range resolutions. By using the special case that transmitter and receiver are both boresight, with same altitude and module of velocity, it is demonstrated that resolutions mainly depends on two factors: bistatic angle $\beta$ and angle between $V_T$ and $V_R$ ($\phi_\omega$). Too large $\beta$ and $\phi_\omega$ can significantly deteriorate the resolvability of BiSAR system. Some other factors which also play key roles in the performance of resolution, like the ratio of transmitter’s and receiver’s slant range, the ratio of transmitter’s and receiver’s module of velocity, and the squint angle of transmitter and receiver respectively, will be the future research problem.

References


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