Switching Equalization Algorithm Based on a New SNR Estimation Method

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Abstract—It is well-known that turbo equalization with the max-log-map (MLM) rather than the log-map (LM) algorithm is insensitive to signal to noise ratio (SNR) mismatch. As our first contribution, an improved MLM algorithm called scaled max-log-map (SMLM) algorithm is presented. Simulation results show that the SMLM scheme can dramatically outperform the MLM without sacrificing the robustness against SNR mismatch. Unfortunately, its performance is still inferior to that of the LM algorithm with exact SNR knowledge over the class of high-loss channels. As our second contribution, a switching turbo equalization scheme, which switches between the SMLM and LM schemes, is proposed to practically close the performance gap. It is based on a novel way to estimate the SNR from the reliability values of the extrinsic information of the SMLM algorithm.

Index Terms—Scaled max-log-map (SMLM), SNR estimation, soft information, switching equalization scheme, turbo equalization.

1. Introduction

In modern mobile communication systems, in order to obtain minimum bit error rate, the symbol log-map (LM) algorithm is preferred [1]. On the other hand, it is sensitive to signal to noise ratio (SNR) estimation errors. Although some suboptimum algorithms, such as max-log-map (MLM) and soft output viterbi algorithm (SOVA), are robust to SNR mismatch [2][4], there is a wide performance gap between the LM turbo detector and these simple algorithms on high-loss channel [2].

For the reduced-complexity algorithms such as the MLM and SOVA with over-estimated extrinsic information, the technique of scaling the extrinsic information by a constant factor is a simple and effective way to improve their performance. Its applications in decoding the low-density parity-check codes (LDPC) codes, the turbo codes and the serial concatenated convolutional codes (SCCC) have been reported in [5]-[8]. However, it seems that there is no related result in the turbo equalization application.

In order to improve the performance of LM with SNR mismatch, one may attempt to estimate the channel SNR more accurately. As it is well known to all, maximum likelihood (ML) estimation method aided by pilot symbols performs closest to CRB bound[9], but in general cases, a great deal of pilots are required in this way. On the other hand, based on the fact that MLM and scaled max-log-map (SMLM) are insensitive to SNR offset, it can be considered that the channel SNR is estimated through MLM or SMLM, so that LM may be used in next frames.

In [2] and [10], an online SNR estimation method is presented, where the hard-decision output of MLM decoder is adopted. Then, the estimated SNR is used in next consecutive frames. Thus, one can not only make full use of the advantages of LM algorithm, but also solve the problem of sensitivity to SNR mismatch. And yet, the performance of MLM may be very poor for some frames in the case of at low SNR and over high-loss channels, hence the estimated SNR might be very inaccurate, which consequently deteriorates the performance of LM.

In this paper, a new reliability-based SNR estimation method is proposed. It is based on the fact that the ratio of mean square of the soft output extrinsic information from the SMLM decoder over its variance (MSVR) is independent of channel SNR offset. Using this new method, we further present a new turbo equalization algorithm, i.e., the switching equalization scheme. It not only performs close to LM with known channel SNR, but also insensitive to SNR mismatch.

The remainder of this paper is organized as follows. We briefly introduce SMLM algorithm and show why it does not require SNR knowledge in Section 2. And a new SNR estimation method is presented in the next section. In Section 4, a new turbo equalization algorithm is described in details. The simulation results are provided in Section 5, and the final part is the conclusion.

2. SMLM Iterative Equalization and Effect on SNR Mismatch

Fig. 1 is the principle block diagram of the turbo receiver scheme of a serially concatenated system composed of the
encoder and the inter-symbol interference (ISI) channel. The soft-in soft-out (SISO) equalizer observes channel values \( r \) and then provides the a posteriori probability log likelihood ratio (LLR) \( L_{E}^{\text{pos}} \) for all coded bits using the a priori information \( L_{E}^{\text{pri}} \). For the first iteration we set \( L_{E}^{\text{pri}} \) to be zero.

The extrinsic information \( L_{E}^{\text{ext}} \) is obtained by subtracting the a priori information from the a posteriori LLRs. After deinterleaving, the extrinsic information is passed to the decoder as priori values \( L_{D}^{\text{pri}} \). The decoder provides the a posteriori values for both the information bits and check bits \( L_{D}^{\text{pos}} \) using the a priori information provided by the equalizer. Again, the extrinsic information \( L_{E}^{\text{ext}} \) is formed after eliminating the effect of the a priori information. After scaling by a constant \( \alpha \) not more than 1 and interleaving, this extrinsic information is delivered to the equalizer as the a priori information. The iterative process continues until the stopping condition is satisfied. Note that MLM algorithm is always used in both the equalizer and the decoder. If \( \alpha = 1 \), Fig. 1 is actually the block diagram of MLM turbo detector.

![Fig. 1. The block diagram of the iterative SMLM equalizer.](image)

The input alphabet set is assumed binary \( \{+1,-1\} \). For MLM equalization algorithm, the a posteriori LLR values can be calculated as follows:

\[
L_{E}^{\text{pos}}(x_k) = \max_{(s',s),x_k=1} (\alpha_{k-1}(s') + \gamma_k(s',s) + \beta_k(s)) - \max_{(s',s),x_k=1} (\alpha_{k-1}(s') + \gamma_k(s',s) + \beta_k(s))
\]

(1)

where \( s' \) and \( s \) denote the channel states at \( k-1 \) and \( k \) instant respectively. \( \alpha_k(s') \) and \( \beta_k(s) \) may be derived recursively by:

\[
\alpha_k(s) = \max_{s'} (\alpha_{k-1}(s') + \gamma_k(s',s))
\]

(2a)

\[
\beta_k,s(x') = \max_{s} (\beta_k(s) + \gamma_k(s',s))
\]

(2b)

Thus, \( \gamma_k(s',s) \) is the sole value to be measured. For the equalizer, it can be obtained by:

\[
\gamma_k(s',s) = -\frac{1}{2\sigma^2_{eq}} \left[ \sum_{i=0}^{L-1} h_i x_{k-i} \right]^2 + \frac{1}{2} \sum_{i=0}^{L-1} L_{E}^{\text{pos}}(x_i)
\]

(3)

where \( h = [h_0, h_1, \cdots, h_{L-1}] \) is the ISI channel coefficient vector, \( \sigma^2_{eq} \) is the estimated noise variance. Assuming the true channel noise power of \( \sigma^2_{ch} \), leads to

\[
\sigma^2_{eq} = P\sigma^2_{ch} = 10\left(\frac{1}{10}\left(\frac{f_0}{f_c}\right)\right)\sigma^2_{ch}
\]

(4)

In the case of no SNR mismatch between the channel and the equalizer, let the a posteriori LLRs be provided by the equalizer \( L_{E,\text{ideal}}^{\text{pos}} \), it is not difficult to derive following result in terms of (1)-(4):

\[
L_{E}^{\text{pos}} = \frac{1}{P} L_{E,\text{ideal}}^{\text{pos}}
\]

(5)

For SMLM algorithm, only the second term in (3) is multiplied by an extra factor \( \alpha \), which will not change the form of (5). Due to \( P > 0 \), it is clear from (5) that the multiplication of the output LLR by \( 1/P \) does not affect the decision on each bit, but the reliability value of the soft output can vary. On the other hand, as it is well known, it has better performance for soft-decision decoding than hard-decision detection. So it is necessary to choose other parameters to use soft output information to estimate the channel SNR. Fortunately, we note that the MSVR of soft output extrinsic LLRs is insensitive to channel SNR offset, while the soft output extrinsic LLRs are correlative with channel SNR offset, but their MSVR is only the function of true channel SNR.

3. SNR Estimation Algorithm

It can be seen from above that the soft output information from both MLM and SMLM can be used to estimate channel SNR. The results from [5]-[8] have shown SMLM can effectively improve the performance of MLM for an iterative decoder. In this paper, we can see that SMLM can also outperform MLM in turbo detector. As seen from Fig. 2, over Proakis C channel[11] and after 8 iterations, the bit error rate (BER) of SMLM turbo detector is about \( 2 \times 10^{-3} \) at \( E_b/N_0 = 5 \) dB, although it is still inferior to \( 2 \times 10^{-4} \) of ideal LM, it is obviously lower than \( 6.5 \times 10^{-2} \) of MLM. Note that the optimal scaling factor \( \alpha \) is obtained by extensive simulations. In simulation, binary phase shift keying (BPSK) modulation is assumed, recursive system convolutional (RSC) codes are selected similar to [2], with the generator polynomial \([1,7/5]_6\) corresponding to the memory length \( M = 2 \).

Obviously, it is more rational to use the extrinsic information of SMLM to estimate channel SNR. As can be
seen from (1)-(6), \( \text{MSVR}_{\text{LLR}} \) is only the function of channel SNR (denoted by \( \text{SNR}_c \)), scaling factor \( \alpha \) and the number of iterations \( n \), i.e., (6) can be simplified as:

\[
\text{MSVR}_{\text{LLR}} = f(\text{SNR}_c, \alpha, n).
\] (7)

Because of the complexity of MLM algorithm, unfortunately, it is difficult to write out the analytic expression of (8). But then, for a time-invariant channel, we can obtain the average \( \text{MSVR}_{\text{LLR}} \) by simulation for all the convergent frames using SMLM at a fixed \( \text{SNR}_c \) in advance. Then a table can be established for different \( \text{SNR}_c \). Through this look-up table, the true channel SNR can be obtained according to the estimated \( \text{MSVR}_{\text{LLR}} \) of each converged frame during decoding. As an example, when the interleaving size is 4096, the partial set of pairs of \((\text{SNR}_c, \text{MSVR}_{\text{LLR}})\) in dB is \{(4.5, 8.35), (5, 10.39), (5.5, 11.25), (6, 12.02)\} over Proakis channel C, their corresponding standard normalized derivation are 0.1377, 0.1172, 0.1119 and 0.1045, respectively, which are very small.

4. Switching Turbo Equalization Algorithm

Using the proposed SNR estimation algorithm, we further present a new equalization scheme called switching turbo equalization. Let’s define some parameters before describing the concrete algorithm.

\( N_{\text{max}} \): The maximum number of iterations;
\( \text{MSVR}_i \): The MSVR of LLR as (6) at the end of \( i \)th iteration;
\( \text{SNR}_c \): The true channel SNR;
\( \text{MSVR}_D \): The enhancement threshold of LLR’s MSVR between adjacent two iterations used to judge the convergence behavior of SMLM;
\( \text{MSVR}_t \): The threshold used to judge the convergence of a frame using SMLM;
\( N_m \): The maximum number of frames consecutively adopting the LM algorithm;
\( n_f \): Counter used to record the number of frames consecutively adopting the LM algorithm;
\( \text{Alg}_\text{Flag} \): Indicate the algorithm of each frame, denotes SMLM if \( \text{Alg}_\text{Flag} = 0 \), LM otherwise.

Throughout the algorithm, the hard-decision aid (HDA) is used for the stopping criteria and to judge the convergence behavior of iterations. The new scheme is:

\[ \text{Input}: N_{\text{max}}, \text{MSVR}_D, \text{MSVR}_t, \text{and } N_m; \]

\[ \text{Step 1: Init. } \text{Alg}_\text{Flag} = 0, \text{MSVR}_i = -1, \text{MSVR}_t = 0; \]

\[ \text{Step 2: For a new frame, perform SMLM algorithm with HDA,} \]

\[ \text{If this frame is convergent, Do} \]

\[ \text{Calculate } \text{MSVR}_{\text{LLR}}; \]
MSVR_{LLR} \rightarrow SNR_c \text{ through looking up table;}
Set Alg_Flg = 1 \text{ and } n_f = 0;
End If
Step 3: If SNR_c = -1, Go back Step 2; End If
Step 4: For a next frame, Perform LM algorithm with HDA, n_f = n_f + 1,
If n_f \leq N_f, Continue;
Else Set Alg_Flg = 0;
End If
Step 5: For a next frame, Perform SMLM algorithm, i = 1
While i < N_{max}, Do
Calculate MSVR_i;
If MSVR_{i} - MSVR_{i-1} \leq MSVR_{i0} and MSVR_{i} < MSVR_{i},
Set Alg_Flg = 1, Go back Step 4;
Else continue;
End If
After stopping iterating, If the frame is convergent, Do
Calculate MSVR_{LLR};
MSVR_{LLR} \rightarrow SNR_c \text{ through looking up table;}
Set Alg_Flg = 1 and n_f = 0; Go back Step 4;
Else Go back Step 5;
End If
i = i + 1;
End while

5. Simulation Results

In this section, we present the simulation results of our algorithm mentioned above. For comparison, the results with ML SNR estimation based on TxDA are also provided. To avoid the concrete frame structure, the estimated SNR is generated by its probability distribution model in ML estimation[9].

To demonstrate the performance of our scheme, we have chosen Proakis C channel with h = \{0.227, 0.46, 0.688, 0.46, 0.227\} and the interleaving size 4096. In simulation, we set N_{max} = 10 and N_e = 4, and always the same algorithm is used in both the equalizer and the decoder. In addition, it is verified by simulation that MSVR_{i0} = 0.04 and MSVR_{i} = 2 are suitable choices.

The ML estimation method of SNR is based on TxDA. To save simulation time, we adopt two RSC codes with small memory length as M = 2 and M = 3. Their generator polynomial are [1,7/5]_8 and [1,17/15]_8 respectively.

Under the conditions above, Fig. 4 shows that when M = 2 and E_b/N_0 \geq 5 dB, our SNR estimation scheme can evaluate the true SNR very exactly, while in low E_b/N_0 region, there is a little loss. This reason is that in bad-channel condition, the convergent probability of SMLM algorithm within the max number of iterations become smaller, so that the outdated estimated SNR is used by more frames. Even so, we can see that at the BER of 10^{-3}, the performance loss is not more than 0.1 dB compared to LM without SNR mismatch.

It is also found from Fig. 4 that our scheme performs almost the same as the one with ML SNR estimation using 50 pilot symbols. In other words, the reliability-based SNR estimation method can at least save 50 pilot symbols in the case of M = 2.

Fig. 4. Performance of switching equalization with interleaving size 4096, over Proakis channel C, N_{max} =10, M=2.

The performance curves for M = 3 are given in Fig. 5. It shows that the larger the memory length of encoder is, the higher precision of SNR estimation is required for iterative detector. Equivalently, for ML estimation, it needs more pilots. At BER = 10^{-3}, even if 100 pilots are used, the performance of ML estimation is still about 0.2 dB worse than that of LM with ideal SNR. Meanwhile, the loss of our new scheme is only less than 0.1 dB. That is to say, for larger memory size, our scheme can also perform close to LM with known SNR, and save more pilots.

Fig. 5. Performance of switching equalization with interleaving size 4096, over Proakis channel C, N_{max} =10, M=3.
6. Conclusions

SMLM turbo equalization algorithm is not only insensitive to SNR offset, but has better performance than MLM. But comparing with LM without SNR error, the performance loss is still large. Fortunately, we find through analysis that for converged frames, the MSVR of the soft output extrinsic information from SMLM decoder is only the function of channel true SNR. It is the basis that we propose a reliability-based SNR estimation method and switching turbo equalization algorithm. The new schemes obtain good BER performance. Moreover, it solves the issue of sensitivity to SNR mismatch. Again, it does not need any pilots. The simulation results verify the validity of our schemes.

References


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