Abstract—This paper proposes an impulsive control scheme for chaotic systems consisting of Van der Pol oscillators coupled to linear oscillators (VDPL) based on their Takagi-Sugeno (T-S) fuzzy models. A T-S fuzzy model is utilized to represent the chaotic VDPL system. By using comparison method, a general asymptotical stability criterion by means of linear matrix inequality (LMI) is derived for the T-S fuzzy model of VDPL system with impulsive effects. The simulation results demonstrate the effectiveness of the proposed scheme.

Index Terms—Impulsive control, T-S fuzzy model, chaos, Van der Pol oscillator.

1. Introduction

Controlling chaotic systems has been widely studied due to undesirable or harmful chaotic behaviors under many circumstances during the last decade. Many methods on chaos control have been proposed and investigated which include linear or nonlinear feedback control, adaptive control, impulsive control, and etc. [1]-[6] At the same time, since impulsive control can stabilize chaotic systems by using only small control impulses, it has been studied by many researchers. Impulsive control has been demonstrated to be an effective and attractive control method to stabilize linear and nonlinear systems, especially to stabilize various chaotic systems [1], [4], [7], [8]. Furthermore, Chaotic systems can usually be represented by the T-S fuzzy model. The T-S fuzzy model is described by fuzzy IF-THEN rules where the consequent parts represent local linear models for nonlinear systems [8]. Therefore, linear theory can be applied to analyze chaotic systems by means of T-S fuzzy models at a certain domain [9], [10].

As it is well known, the Van der Pol oscillator, which is a typical nonlinear chaotic system has many interesting features and numerous applications. Recently, a coupled Van der Pol oscillators was introduced in [11], which was derived from a new electronic circuit modelling the dynamics of a Van der Pol oscillator coupled to a linear oscillator (called as VDPL). The VDPL system is described as follows:

\[
\begin{align*}
\frac{d}{dt} x_2 &= x_2 \\
\frac{d}{dt} x_2 &= \varepsilon \left( 1 - x_1^2 \right) x_2 - x_1 + \alpha (x_4 + \omega_2^2 x_3 - \lambda x_1) \\
\frac{d}{dt} x_3 &= x_4 \\
\frac{d}{dt} x_4 &= -\varepsilon x_4 - \omega_2^2 x_3 + \lambda x_1.
\end{align*}
\]

With the parameters \( \varepsilon = 3.872 \), \( \varepsilon = 0.000645 \), \( \lambda = 9.12 \), \( \alpha = 0.547 \), and \( \omega_2^2 = 5 \), the system (1) shows a chaotic behavior [11],[12]. More recently, the problem of stabilization and synchronization of chaotic VDPL systems was investigated by using adaptive control in [12].

In this paper, we further study an impulsive control scheme for T-S fuzzy model-based chaotic VDPL systems. A general asymptotical stability criterion is given to stabilize chaotic VDPL systems via impulsive control. The simulation results show the effectiveness of our control scheme.

Throughout this paper, \( \mathbf{A} > 0 \) or \( \mathbf{A} < 0 \) means \( \mathbf{A} \) is a symmetrical positive or negative definite matrix. \( i^+, i^- \), and \( \mathbb{N} \) stand for the set of all positive real numbers, nonnegative real numbers, and the set of natural numbers, respectively. \( \| \mathbf{x} \| \) denotes the Euclidian norm of vector \( \mathbf{x} \), \( (\cdot)' \) denotes the derivative of \((\cdot)\), \( \lambda_{\min} (\mathbf{P}) \) and \( \lambda_{\max} (\mathbf{P}) \) mean the minimal and maximal eigenvalues of matrix \( \mathbf{P} \), respectively.

2. Problem Formulation

For (1), we can construct the fuzzy model as follows.

Rule \( i \): If \( z_i(t) \) is \( M_{i1} \), \( \mathbf{L} \), and \( z_p(t) \) is \( M_{i2} \), then

\[
\mathbf{x}(t) = \sum_{i=1}^{L} h_i(t) \mathbf{A}_i \mathbf{x}(t) \tag{2}
\]
where \( r \) is the number of fuzzy implications, and
\[
h_i(t) = \frac{\omega_i(t)}{\sum_{i=1}^{\rho} \omega_i(t)}, \quad \omega_i(t) = \prod_{i=1}^{p} M_{\rho}(z_i(t))
\]
and \( M_{\rho}(z_i(t)) \) is the grade of membership of \( z_i(t) \) in \( M_{\rho} \). Of course, \( h_i(t) \geq 0 \) and \( \sum_{i=1}^{r} h_i(t) = 1 \). Note that system (2) can locally represent system (1).

An impulsive control T-S fuzzy model of the VDPL system is described by
\[
\begin{align*}
    x(t) &= \sum_{i=1}^{r} h_i(t) A_i x(t), & \forall t \geq 0, t \neq t_k \\
    \Delta x(t) &= C_i x(t), & t = t_k, k \in \mathbb{Y}
\end{align*}
\]
where \( C_i \in \mathbb{C}[a, b] \) denotes the incremental change of the state at time \( t_k, 0 < t < t_k < L < t_k < L \), \( t_k \to \infty \) as \( t \to \infty \), and \( \Delta x(t) = x(t^+_k) - x(t) \), \( x(t^+_k) = \lim_{t \to t^+_k} x(t), k \in \mathbb{Y} \).

For the following discussion, we first give some definitions and lemmas as follows.

Definition 1. Let \( V : \mathbb{R}^n \to \mathbb{R}^+ \) be continuous in \( \mathbb{R}^n \) and locally Lipschitzian in \( x \).

Definition 2. For \( (t, x) \in (t_{k-1}, t_k) \times a^+ \), we define the right upper derivative of \( V(t, x) \) is
\[
D^rV(t, x) = \limsup_{s \to 0^+} \frac{1}{s}[V(t + s, x + s f(t, x)) - V(t, x)].
\]

Definition 3. (Comparison system) Let \( V \in \mathcal{V}_0 \) and assume that
\[
\begin{align*}
    D^rV(t, x) &\leq g(t, V(t, x)), & t \neq t_k \\
    V(t, x + C_i x(t)) &\leq \psi_i(V(t, x)), & t = t_k
\end{align*}
\]
where \( g : \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^+ \) is continuous and \( \psi_i : \mathbb{R}^+ \to \mathbb{R}^+ \) is nondecreasing. Then the comparison system of (3) is
\[
\begin{align*}
    \alpha' &= g(t, \alpha), & t \neq t_k \\
    \alpha(t_k^+) &= \psi_i(\alpha(t_k)), \quad \alpha(t_k^+) \geq 0
\end{align*}
\]
\[
(5)
\]
Lemma 1.\textsuperscript{[12]} Assume that

1. For \( V : x^+ \to \mathbb{R}^+ \), where \( S_\rho = \{ x | x^+ \} \) with \( \rho > 0 \) and \( V \in \mathcal{V}_0 \), \( D^rV(t, x) \geq g(t, V(t, x)) \), \( t \neq t_k \);
2. There exist a \( \rho_0 > 0 \) such that \( x \in S_{\rho_0} \) implies that \( x + C_i x(t) \in S_{\rho_0} \) for all \( k \) and \( V(t, x + C_i x(t)) \leq \psi_i(V(t, x)) \) for \( t = t_k \) and \( x \in S_{\rho_0} \);
3. \( \beta(||x||) \leq V(t, x) \leq \alpha(||x||) \) on \( x^+ \in S_\rho \), where \( \alpha(x) \in \mathbb{R}^+ \) and \( \beta(x) \in \mathbb{R}^+ \) (class of continuous strictly increasing functions: \( \alpha \in \mathbb{C}[a, b] \), \( \alpha(0) = 0 \)).

Then the stability of the trivial solution of comparison system (5) implies the corresponding stability of the trivial solution of (3).

Lemma 2.\textsuperscript{[11]} Let \( g(t, \alpha) = \lambda(t)\alpha, \lambda \in C[0, + \infty) \) and \( \psi_i(\alpha) = d_i \alpha, d_i \geq 0 \) for all \( k \), then the trivial solution of (3) is asymptotically stable if conditions
\[
\lambda(t_{k+1}) + \ln(\gamma d_k) \leq \lambda(t_k), \quad \gamma > 1
\]
\[
(6)
\]
\[
\lambda(t_k^+) \geq 0
\]
\[
(7)
\]
are satisfied.

3. Main Results

In this section, we shall give an asymptotic stability criterion of impulsive system (3).

Theorem 1. The origin of impulsive system (3) is asymptotically stable if there exist scalars \( \lambda_i \geq 0, \gamma > 1 \), and \( \lambda_i \geq 0, k \in \mathbb{Y} \), and the following condition holds:
\[
\lambda_i(t_{k+1}) - \lambda_i(t_k) \leq -\ln(\gamma d_k)
\]
\[
(8)
\]
where \( \lambda_i \) is the largest one of \( \lambda_i, i = 1, 2, L \), whereas \( \lambda_i \) is the largest eigenvalue of matrix \( A_i + A_i^T \), \( i = 1, 2, \ldots, r \). And \( d_k \) is the largest eigenvalue of matrix \( (I + C_i)(I + C_i^T) \) for \( k \in \mathbb{Y} \), where \( I \) is an identity matrix.

Proof. Consider the Lyapunov function \( V(x(t)) = x^T(t)x(t) \). When \( t \neq t_k \), the right upper derivative of \( V(x(t)) \) is
\[
D^rV(x(t)) = [x(t)]^T x(t) + x^T(t)x(t) = \sum_{i=1}^{r} h_i(t)[x^T(t)(A_i + A_i^T)x(t)]
\]
\[
(9)
\]
Then we have
\[
D^rV(x(t)) \leq \lambda_i V(x(t)) \leq \lambda_i V(x(t))
\]
\[
(10)
\]
Hence, the conditions of Lemma 1 are satisfied with \( g(t, \alpha) = \lambda_i \alpha \) and \( \psi_i(\alpha) = d_i \alpha \). It followed by Lemma 1 that the asymptotic stability of impulsive system (3) is implied by that of the following comparison system:
\[
\begin{align*}
    \alpha' &= \lambda_i \alpha, & t \neq t_k \\
    \alpha(t_k^+) &= d_i \alpha(t_k), \quad \alpha(t_k^+) \geq 0
\end{align*}
\]
\[
(11)
\]
According to (6) and Lemma 2, the origin of system (3) is asymptotically stable. This concludes the proof.

Remark 1. If system (3) is stabilizable, we can stabilize
the VDPL system (1) via impulsive control.

Remark 2. According to the definition of $d_k$ and Schur complement, the inequality $(I + C_k^T)(I + C_k) - d_k I \leq 0, k \in \mathbb{N}$ is equivalent to the following linear matrix inequality (LMI):

$$
\begin{bmatrix}
-d_k I & I + C_k^T \\
I & -C_k
\end{bmatrix} \leq 0, \quad k \in \mathbb{N}.
$$

(12)

Fig. 1. Response of impulsive controlled VDPL system.

4. Numerical Simulation

Setting the VDPL system (1) with the parameters $\varepsilon_1 = 3.872$, $\varepsilon_2 = 0.000645$, $\lambda = 9.12$, $\alpha = 0.547$, and $\omega^2 = 5$, the VDPL system (1) exhibits chaotic activity, as reported in [10] and [11]. Assume that $x_i \in [-d, d]$, we set $d = 10$ in our simulation, it can be verified that the VDPL system can be exactly represented by the following T-S fuzzy model.

Rule $i$ ($i = 1, 2$): IF $x_i$ is $M_i$ THEN $x'_i = A_i x_i, t$,

where

$$
x = [x_1, x_2, x_3, x_4]^T
$$

$$
A_1 = \begin{bmatrix}
0 & 1 & 0 & 0 \\
-(1 + \alpha \lambda) & \varepsilon_1 (1 - d^2) & \alpha \omega^2 & \alpha \\
0 & 0 & 0 & 1 \\
\lambda & 0 & -\omega^2 & -\varepsilon_2
\end{bmatrix}
$$

$$
A_2 = \begin{bmatrix}
0 & 1 & 0 & 0 \\
-(1 + \alpha \lambda) & \varepsilon_1 & \alpha \omega^2 & \alpha \\
0 & 0 & 0 & 1 \\
\lambda & 0 & -\omega^2 & -\varepsilon_2
\end{bmatrix}
$$

$$
M_i(x_i) = \frac{x_i^2}{d^2}, \quad M_2(x_i) = 1 - M_i(x_i).
$$

Then the VDPL system can be represented as

$$
x'(t) = \sum_{i=1}^{2} M_i(x_i(t)) A_i x_i(t).
$$

(13)

According to Theorem 1, we can obtain $\lambda = 13.2830$. For computational convenience, we let $t_k = 0.04k$ for $k \in \mathbb{N}$, so $C_k = C$ is a scalar matrix and $d_k = d$. Let $\gamma = 1.01$, and $\lambda = 0.5820$ is calculated from (8). By solving the LMI (12), we have $C = \text{diag}[-0.2371, -0.2371, -0.2371, -0.2371]$. The simulation results in Fig. 1 show that the origin of the impulsive controlled VDPL system (1) is asymptotically stable.

5. Conclusions

In this paper, an impulsive control scheme is presented for chaotic VDPL systems consisting of a Van der Pol oscillator coupled to a linear oscillator based on their T-S models. A simple asymptotically stable criterion is derived and can be conveniently applied to design impulsive controllers for the VDPL systems. Numerical simulations show the effectiveness of the impulsive control scheme.

References


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