Modeling Piezoelectric Interfacial Wave Near an Imperfect Interface

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Abstract The interface wave propagating along an imperfect interface between two piezoelectric half spaces is derived firstly. The wave equations based on the interface modeled, called “spring model”, are presented. The micro-scale structures of the interface for connecting the spring constant with the interface micro-structures are examined. For some simple interface micro-structure, exact dynamic solution is available, and the spring constant is obtained by comparing solutions. For the complex micro structures, it remains as a challenge of micro-mechanics modeling to connect the “spring constant” and micro-structure.

Key words piezoelectric waves; self-consistent scheme; imperfect interface

1 Imperfect Interface Modeled by a Spring Model

The configuration under our concern is given in Fig.1. The ceramics are both poled along the ±X3 directions. We are interested in the possibility of a wave traveling near the interface between the two ceramics. The interface waves of this type in the linear elastic and piezoelectric materials for the perfect interface were studied by Refs.[10-14]; while the imperfect interface remains an un-touch research area.

The governing equations are\[^{[11]}\]

\[\begin{align*}
\tau_A \nabla^2 u_A &= \rho_A \frac{d^2}{dx^2} u_A, \quad \nabla^2 \psi_A = 0, \quad x_2 > 0 \quad (1a) \\
\tau_B \nabla^2 u_B &= \rho_B \frac{d^2}{dx^2} u_B, \quad \nabla^2 \psi_B = 0, \quad x_2 < 0 \quad (1b)
\end{align*}\]

where the subscripts A and B indicate ceramic A and ceramic B. For an interface wave, we require

\[u_A, \quad \psi_A \rightarrow 0, \quad x_2 \rightarrow +\infty \quad (2a)\]
For \( x_2 > 0 \), the solutions of Eq. (15a) that satisfy the
Eq. (16a) can be written as

\[
\begin{align*}
\psi_A &= U_A \exp(\eta_A x_2) \cos(\xi_A x_1 - \omega t) \\
\psi_B &= U_B \exp(\eta_B x_2) \cos(\xi_B x_1 - \omega t)
\end{align*}
\]

where

\[
\eta_A^2 = \xi_A^2 - \frac{\rho_A \omega^2}{c_A^2} = \xi_A^2(1 - \frac{\nu_A^2}{\nu_B^2}) > 0
\]  

and

\[
\nu = \frac{\omega}{\xi}, \quad \nu_A^2 = \frac{c_A^2}{\rho_A}
\]

Similarly, for \( x_2 < 0 \), the solutions of Eq. (1b) that satisfy Eq. (2b) are

\[
\begin{align*}
\psi_A &= U_A \exp(\eta_B x_2) \cos(\xi_B x_1 - \omega t) \\
\psi_B &= U_B \exp(\eta_A x_2) \cos(\xi_A x_1 - \omega t)
\end{align*}
\]

where

\[
\eta_B^2 = \xi_B^2 - \frac{\rho_B \omega^2}{c_B^2} = \xi_B^2(1 - \frac{\nu_B^2}{\nu_A^2}) > 0
\]  

and

\[
\nu_B^2 = \frac{c_B^2}{\rho_B}
\]

When the interface is a grounded electrode, at \( x_1 = 0 \) we have,

\[
T_\alpha = T_\beta = T, \quad T = K(u_A - u_B)
\]

\[
\psi_A + \frac{e_A}{\varepsilon_A} u_A = 0, \quad \psi_B + \frac{e_B}{\varepsilon_B} u_B = 0
\]

where \( K \) is the “spring constant” describing the interface imperfection. An infinite \( K \) value means a perfect interface, where displacement is continuous across the interface. While a zero value of the \( K \) means that the interface is the mechanically free with zero traction.

Substitution of Eq. (3) and Eq. (6) into Eq. (9) results in a system of linear homogenous equations for \( U_\alpha, \Psi_\alpha, U_\beta \) and \( \Psi_\beta \). For nontrivial solutions, the determinant of the coefficient matrix has to vanish, which yields

\[
(\tau_\alpha \eta_A - \frac{e_A^2}{\varepsilon_A}) (\tau_\alpha \eta_B - \frac{e_B^2}{\varepsilon_B}) = -K (\tau_\alpha \eta_A - \frac{e_A^2}{\varepsilon_A}) (\tau_\alpha \eta_B - \frac{e_B^2}{\varepsilon_B}) + (\tau_\beta \eta_A - \frac{e_A^2}{\varepsilon_A}) (\tau_\beta \eta_B - \frac{e_B^2}{\varepsilon_B})
\]

\[
(\tau_\alpha \eta_A - \frac{e_A^2}{\varepsilon_A}) (\tau_\alpha \eta_B - \frac{e_B^2}{\varepsilon_B}) = -K (\tau_\alpha \eta_A - \frac{e_A^2}{\varepsilon_A}) (\tau_\alpha \eta_B - \frac{e_B^2}{\varepsilon_B}) + (\tau_\beta \eta_A - \frac{e_A^2}{\varepsilon_A}) (\tau_\beta \eta_B - \frac{e_B^2}{\varepsilon_B})
\]

Eq. (10) determines the speed of the interface wave. Some special cases are discussed as follows.

1) \( K = 0 \). The interface is mechanically free. The half-space \( A \) and \( B \) deform independently with electric potential grounded at the free surface. Then, the speed of the Bleustein-Gulyaev wave within ceramic \( A \) and ceramic \( B \) respectively \([11,13]\) can be obtained by

\[
v^2 = v_A^2[1 - (\frac{\varepsilon_A^2}{\varepsilon_A^2})^2], \quad v^2 = v_B^2[1 - (\frac{\varepsilon_B^2}{\varepsilon_B^2})^2]
\]

2) \( K \to \infty \). The interface is perfectly bonded. Setting the second term of right-hand-side of Eq. (10) into zero, we have

\[
(\tau_\alpha \eta_A - \frac{e_A^2}{\varepsilon_A}) (\tau_\alpha \eta_B - \frac{e_B^2}{\varepsilon_B}) = -2K \quad (\tau_\alpha \eta_A - \frac{e_A^2}{\varepsilon_A}) (\tau_\alpha \eta_B - \frac{e_B^2}{\varepsilon_B}) = 0
\]

\[
e_A = e_B = \varepsilon_A = \varepsilon_B = e
\]

\[
\rho_A = \rho_B = \rho
\]

\[
v_A = v_B = v
\]

Eq. (10) is rewritten as

\[
(\tau_\alpha \eta_A - \frac{e_A^2}{\varepsilon_A}) (\tau_\alpha \eta_B - \frac{e_B^2}{\varepsilon_B}) = -2K \quad (\tau_\alpha \eta_A - \frac{e_A^2}{\varepsilon_A}) (\tau_\alpha \eta_B - \frac{e_B^2}{\varepsilon_B}) = 0
\]

2 Wave near the Electrode Layer

Two piezoelectric half-spaces of polarized ceramics with a \( 2h \) metal layer between them (see Fig. 2) are considered. The ceramics are poled in either the \( x_3 \) direction or its opposite. Acoustic waves in the two half-spaces can be coupled by the displacement field in the metal layer. We consider surface waves in the half-spaces near \( x_2 = \pm h \).
require that
\[ u_A, \ \psi_A \rightarrow 0, \ x_2 \rightarrow +\infty \quad (16a) \]
\[ u_B, \ \psi_B \rightarrow 0, \ x_1 \rightarrow -\infty \quad (16b) \]

For \( x_2 > h \), the solutions of Eq. (1a) that satisfy Eq. (2a) can be written as
\[ u_A = U_A \exp(-\eta_A(x_2 - h))\cos(\xi x_1 - \omega t) \quad (17a) \]
\[ \psi_A = \Psi_A \exp(-\xi(x_2 - h))\cos(\xi x_1 - \omega t) \quad (17b) \]
where \( U_A \) and \( \Psi_A \) are undetermined constants,
\[ \eta_A^2 = \xi^2 - \frac{\rho_A}{\varepsilon_A} = \xi^2(1 - \frac{v_A^2}{v_A^2}) > 0 \quad (18) \]
and
\[ \nu = \frac{\omega}{\xi}, \quad \nu_A = \frac{\varepsilon_A}{\rho_A} \quad (19) \]

For continuity conditions, we need \( T_{23} \) in ceramic \( A \), denoted by \( T_A \):
\[ T_A = \tau_{AA_2} + e_A \psi_{A_2} = \]
\[ -[\tau_{AA_2}, \eta_A U_A \exp(-\eta_A(x_2 - h)) + e_A \xi \Psi_A \exp(-\xi(x_2 - h))\cos(\xi x_1 - \omega t) \quad (20) \]

Similarly, for \( x_2 < -h \), the solutions of Eq. (15c) that satisfy Eq. (16b) are
\[ u_B = U_B \exp(\eta_B(x_2 + h))\cos(\xi x_1 - \omega t) \quad (21a) \]
\[ \psi_B = \Psi_B \exp(\xi(x_2 + h))\cos(\xi x_1 - \omega t) \quad (21b) \]
where \( U_B \) and \( \Psi_B \) are undetermined constants,
\[ \eta_B^2 = \xi^2 - \frac{\rho_B}{\varepsilon_B} = \xi^2(1 - \frac{v_B^2}{v_B^2}) > 0 \quad (22) \]
and
\[ \nu_B^2 = \frac{\varepsilon_B}{\rho_B} \quad (23) \]

For continuity conditions, we need \( T_{23} \) in ceramic \( B \), denoted by \( T_B \):
\[ T_B = \tau_{BB_2} + e_B \psi_{B_2} = \]
\[ [\tau_B \eta_B \ U_B \exp(\eta_B(x_2 + h)) + e_B \xi \Psi_B \exp(\xi(x_2 + h))\cos(\xi x_1 - \omega t) \quad (24) \]

The fields in the metal layer can be written as
\[ u_M = (M_1 \cosh \xi x_2 + M_2 \sinh \xi x_2)\cos(\xi x_1 - \omega t) \quad (25) \]
where \( M_1 \) and \( M_2 \) are undetermined constants. Satisfying Eq. (15b) leads to
\[ \xi^2 = (1 - \frac{v_1^2}{v_M^2})\xi^2, \quad \nu^2 = \frac{\omega^2}{\xi^2}, \quad v_B^2 = \frac{c_M}{\rho_M} \quad (26) \]

For the interface continuity, we need \( T_{23} \) in the metal layer \( M \), denoted by \( T_M \):
\[ T_M = c_M u_{MM_2} = c_M \xi \ [M_1 \sinh \xi x_2 + M_2 \cosh \xi x_2] \cos(\xi x_1 - \omega t) \quad (27) \]

As the interface continuity conditions, we suppose
\[ T_A = T_M, \ \psi_A + \frac{e_A}{\varepsilon_A} U_A = 0, \ u_A = u_M, \ x_2 = h \quad (28a) \]
\[ T_B = T_M, \ \psi_B + \frac{e_B}{\varepsilon_B} U_B = 0, \ u_B = u_M, \ x_2 = -h \quad (28b) \]

Eq. (28) implies that
\[ -[\tau_A \eta_A U_A + e_A \xi \Psi_A] = c_M \xi \ [M_1 \sinh \xi h + M_2 \cosh \xi h] \quad (29a) \]
\[ \psi_A + \frac{e_A}{\varepsilon_A} U_A = 0 \quad (29b) \]
\[ U_A = M_1 \cosh \xi h + M_2 \sinh \xi h \quad (29c) \]
\[ \tau_B \eta_B U_B + e_B \xi \Psi_B = c_M \xi (-M_1 \sinh \xi h + M_2 \cosh \xi h) \quad (29d) \]
\[ \psi_B + \frac{e_B}{\varepsilon_B} U_B = 0 \quad (29e) \]
\[ U_B = M_1 \cosh \xi h - M_2 \sinh \xi h \quad (29f) \]

Eq. (29) can be rearranged into Eq. (30):
For nontrivial solutions the determinant of the coefficient matrix has to vanish, which yields the dispersion relation of the waves.

We examine the special case when the two ceramic half-spaces are of the same material:

\[
\begin{align*}
\bar{e}_A &= \bar{e}_B = \bar{e}, \\
\bar{\rho}_A &= \bar{\rho}_B = \bar{\rho}, \\
v_A &= v_B = v_f
\end{align*}
\]

Then the waves can be separated into symmetric and anti-symmetric ones.

### 2.1 Symmetric Waves

For symmetric waves we consider

\[
\begin{align*}
U_A &= U_B, \\
\Psi_A &= \Psi_B, \\
M &= 0
\end{align*}
\]

for which the last three equations of Eq.(30) become identical to the first three, and the dispersion relation assumes the following simple form:

\[
\frac{\bar{\tau}}{c_m} \tanh(\bar{h}v_f - \bar{K} - \eta) = 0
\]

Eq. (34) can be further written as

\[
(\bar{\tau} - \bar{e} c_m \bar{K}) \cosh \bar{h} + c_m \bar{h} \sinh \bar{h} = 0
\]

where we have introduced the coupling factor of the piezoelectric ceramics:

\[
\bar{K} = \frac{c_m}{v_f}
\]

With Eq.(18) and Eq.(26), Eq.(35) takes the following form

\[
(1 - \frac{v_f^2}{v_m^2})^{1/2} \tanh(\bar{h}v_f(1 - \frac{v_f^2}{v_m^2})) = \frac{\bar{\tau}}{c_m}(\bar{K} - (1 - \frac{v_f^2}{v_m^2}))
\]

Eq.(37) shows that the wave is dispersive. For a long wave approximation, where \(\bar{h}v_f \to 0\), we have

\[
\frac{\bar{\tau}}{c_m}(\bar{K} - (1 - \frac{v_f^2}{v_m^2})) = 0
\]

as the zero-th order approximation, and

\[
\bar{h}v_f(1 - \frac{v_f^2}{v_m^2}) = \frac{\bar{\tau}}{c_m}(\bar{K} - (1 - \frac{v_f^2}{v_m^2}))
\]

as the first order approximation. It is seen that Eq.(38a) leads to a non-dispersive wave; while the first order approximation leads to dispersive wave.

### 2.2 Anti-symmetric Waves

For anti-symmetric waves we consider

\[
U_A = -U_B, \quad \Psi_A = -\Psi_B, \quad M = 0
\]

Then the last three equations of Eq.(30) become identical to the first three, and the dispersion relation assumes the following simple form:

\[
\frac{\bar{\tau}}{c_m} \tanh(\bar{h}v_f - \bar{K} - \eta) = 0
\]

With Eq.(18), Eq.(42) takes the following form:

\[
\frac{\bar{\tau}}{c_m}(\bar{K} - (1 - \frac{v_f^2}{v_m^2})\bar{h} = 0
\]

For the long wave approximation, we have

\[
\frac{\bar{\tau}}{c_m}(\bar{K} - (1 - \frac{v_f^2}{v_m^2})\bar{h} = \frac{1}{3}(1 - \frac{v_f^2}{v_m^2}) = 1
\]

### 2.3 Comparison between Two Formulations

In order to obtain some insight on these formulations, we consider the case when the two ceramic half-spaces are made of the same material. The exact solutions, symmetric and anti-symmetric waves, are given in Section 2.1 and 2.2. The approximate solution based on the spring model is deduced from Eq.(24) by setting material A equals material B. By using the notation given in Eq.(31), we simplify Eq.(24) as

\[
(\bar{\tau} - \bar{e} c_m \bar{K}) \cosh \bar{h} + c_m \bar{h} \sinh \bar{h} = 0
\]

or

\[
\bar{e} c_m \cosh \bar{h} + (\bar{\tau} - \bar{e} c_m \bar{K}) \sinh \bar{h} = 0
\]

Eq.(44) is the lowest order approximation of the anti-symmetric wave. Comparing Eq.(45) with Eq.(39) and Eq.(43), we find that Eq.(45) is the lowest order approximation of
Eq. (39) and Eq. (43) if we set
\[ K = \frac{c_u}{2h} \]  
(46)

3 Concluding Remarks

In the present paper, we introduced an imperfect interface model for the softened interface. The imperfect interface can be derived from various micro-mechanisms. For example, Section 2 provided a mechanism where the softened interface is a third phase material. More rigorous derivation along this line can be found in Ref. [15]. The other mechanisms, for example, a micro-crack damaged interface [16], are also possible reasons for the soft interface. Nevertheless, analytical dynamic solution for the latter mechanism is not available. Therefore, connection of the imperfect interface stiffness and micro-structure is a challenge task for the micro-mechanics modeling.

References


Brief Introduction to Author(s)

XU Li-mei (徐利梅) was born in Sichuan, China, in 1969. She received B.S. and M.S. degree from the University of Electronic Science & Technology of China (UESTC), in 1991 and 1994, respectively, and Ph.D. degree from Nanyang Technological University, Singapore in 2001. After joining UESTC in 2003, her research interests are in the areas of structural vibration, Micro electro-mechanical system modeling and measurement.

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