Synchronization and Parameters Identification of Chaotic Systems via Adaptive Control

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Abstract Based on Lyapunov stability theory, a novel adaptive controller is designed for a class of chaotic systems. The parameters identification and synchronization of chaotic systems can be carried out simultaneously. The controller and the updating law of parameters identification are directly constructed by analytic formula. Simulation results with Chen’s system and Rössler system show the effectiveness of the proposed controller.

Key words chaos; parameters identification; synchronization; adaptive control

Since the idea of synchronization of chaotic systems was introduced by Pecora and Carroll in 1990, more and more attention has been paid to the synchronization of chaotic systems and its applications[1, 2]. All kinds of approaches such as linear or nonlinear feedback synchronization method, adaptive control synchronization method, backstepping control synchronization method, among many others have been proposed[3~9]. Those methods are valid for chaotic systems with known or unknown parameters.

For a class of chaotic systems dependent linearly on unknown parameters, Chen et al. studied the problem of parameters identification and synchronization[10]. With the assumption about controller, a parameters identification and synchronization method has been proposed. However, this assumption seems complicated, the scalar function V must be constructed firstly and the adaptive controller can not be constructed directly. In this paper, a novel adaptive controller is designed, and the controller and adaptive laws of unknown parameters can be constructed directly by analytic formula without more assumption about the systems. Chaos synchronization and parameters identification can be achieved simultaneously by the adaptive controller.

The rest of this paper is organized as follows. In section 1, the problem formulation is presented. The design of the adaptive controller is studied in section 2, the numerical simulation with Chen’s system and Rössler system with unknown parameters have shown the effectiveness of the proposed controller in section 3 and 4. Section 5 gives the conclusion.

1 Problem Formulation

Consider the chaos systems
\[ \frac{dx}{dt} = f(x) + F(x)\theta \] (1)
where \( x \in \mathbb{R}^n \) is the state vector of the system, \( \theta \in \mathbb{R}^n \) is the vector of the system parameters, \( f \in C(R^n, R^n) \) and \( F \in C(R^n, R^{n\times n}) \). Eq.(1) is drive system driving the following response system
\[ \frac{d\alpha}{dt} = f(\alpha) + F(\alpha)\theta \] (2)
which has the same structure as the drive system and \( \alpha \in \mathbb{R}^n \) is the estimate of parameter \( \theta \), which is unknown to the response system. Defining \( e(t) = u(t) - x(t) \) is the state error and \( \beta(t) = a(t) - \theta \) is the parameter estimate error, the goal of the control is to find out a controller \( U \) such that the controlled response system
\[ \frac{d\alpha}{dt} = f(\alpha) + F(\alpha)\alpha + U \] (3)
is synchronous with the drive system, i.e., such that \( \lim_{t \to +\infty} e(t) = 0 \), \( \lim_{t \to +\infty} \beta(t) = 0 \), and the unknown parameters can be identified simultaneously.

Form Eqs.(3) and (1), we have the error system
\[ \frac{de}{dt} = f(u) - f(x) + (F(u) - F(x))\alpha + F(x)\beta + U \] (4)

2 Design of Adaptive Control

In order to achieve the goal of control, we choose
the controller in the form:

$$U = -\Gamma e + f(x) - f(u) - (F(u) - F(x))a$$  \hspace{1cm} (4)$$

where \( \Gamma \) is any positive definite symmetric matrix, and the adaptive law of estimate error is in the form:

$$\frac{d\alpha}{dt} = -F^T(x)e$$  \hspace{1cm} (5)$$

Substituting Eq.(4) into the error system, we have the error system and parameters identification system as

$$\begin{align*}
\frac{de}{dt} &= -\Gamma e + F(x)\beta \\
\frac{d\alpha}{dt} &= -F^T(x)e
\end{align*}$$  \hspace{1cm} (6)$$

**Theorem** The response system in Eq.(3) controlled by the controller in Eq.(4) is synchronous with the drive system in Eq.(1), and the parameter estimate updated by Eq.(5) satisfies

$$\lim_{t \to +\infty} \alpha(t) - \theta = 0$$

**Proof** For Eq.(6), choose Lyapunov function

$$V = \frac{1}{2} e^T e + \frac{1}{2} \beta^T \beta$$

We have

$$\frac{dV}{dt} = e^T \frac{de}{dt} + \beta^T \frac{d\beta}{dt} = e^T (-\Gamma e + F(x)\beta) + \beta^T \frac{d\alpha}{dt} = -e^T \Gamma e + e^T F(x)\beta + \beta^T \frac{d\alpha}{dt} = -e^T \Gamma e \leq 0$$  \hspace{1cm} (7)$$

Therefore, the solution of Eq.(6) in about equilibrium point \( e(t) = 0 \) and \( \alpha = \theta \) is globally uniformly stable. Then \( e(t) \) and \( \alpha(t) \) are globally bounded for \( t \geq 0 \), vector \( e(t) \) is square-integrable by Eq.(7), from Eq.(6), and \( de/dt \) is bounded. By Barbalat’s lemma, we conclude that \( e(t) \to 0 \), as \( t \to +\infty \), that is, \( \lim_{t \to +\infty} e(t) = 0 \) [11].

Moreover, from Eq.(6), we can easily find \( d^2 e/dt^2 \) is bounded, that is \( \|d^2 e/dt^2\| \leq K \), where \( K \) is a positive constant. Suppose \( \lim_{t \to +\infty} de/dt \neq 0 \), then there exists an infinite unbounded sequence \( \{t_n\} \) and \( \varepsilon > 0 \) such that \( \|de(t_n)/dt\| \geq \varepsilon \). For any \( t > t_n \), we have

$$e(t) = e(t_n) + \frac{de(t_n)}{dt}(t-t_n) + 0.5 \frac{d^2 e(t_n)}{dt^2}(t-t_n)^2$$

where \( t \in (t_n, t) \), then

$$\|e(t)\| = \left\| e(t_n) + \frac{de(t_n)}{dt}(t-t_n) + 0.5 \frac{d^2 e(t_n)}{dt^2}(t-t_n)^2 \right\| \geq \left\| \frac{de(t_n)}{dt}(t-t_n) + 0.5 \frac{d^2 e(t_n)}{dt^2}(t-t_n)^2 \right\|$$

Therefore

$$\left\| e(t) \right\| + \left\| e(t_n) \right\| \geq \left\| \frac{de(t_n)}{dt}(t-t_n) + 0.5 \frac{d^2 e(t_n)}{dt^2}(t-t_n)^2 \right\|$$

Taking \( t = t_n + \delta \), then

$$\|e(t)\| + \|e(t_n)\| \geq \delta(e - K/2)$$

Choosing \( \delta = \varepsilon / K \), we have

$$\|e(t)\| + \|e(t_n)\| \geq \frac{\varepsilon^2}{2K}$$

Hence, we get a contradiction with \( e(t) \to 0 \) as \( t \to +\infty \), this implies \( \lim_{t \to +\infty} de/dt = 0 \). In the same way we have \( \lim_{t \to +\infty} de/dt = 0 \). That is \( \lim_{t \to +\infty} de/dt = 0 \).

From Eq.(6), we conclude that \( \lim_{t \to +\infty} \beta(t) = 0 \).

This completes the proof.

**Remark** From the theorem, we can design the adaptive controller by the analytic formulas in Eqs.(4) and (5) more easily and directly than by the method in Ref.[10].

### 3 Simulation with Chen’s Chaotic System

In this section, the Chen’s system is used to demonstrate the effectiveness of the proposed controller.

Consider the Chen’s chaotic system in the following form:

$$\begin{align*}
\frac{dx}{dt} &= \theta_1(y - x) \\
\frac{dy}{dt} &= (\theta_2 - \theta_1)x - xy + \theta_3 y \\
\frac{dz}{dt} &= xy - \theta_1 z
\end{align*}$$  \hspace{1cm} (8)$$

which has a chaotic attractor when \( \theta_1 = 35 \), \( \theta_2 = 28 \), and \( \theta_3 = 3 \). In this case, we have
For the controlled system

\[
\begin{align*}
\frac{du}{dt} &= \alpha_1(t)(v-u) + U_i(t) \\
\frac{dv}{dt} &= (\alpha_2(t) - \alpha_1(t))u - vw + \alpha_2(t)v + U_z(t) \\
\frac{dw}{dt} &= uv - \alpha_3(t)z + U_3(t)
\end{align*}
\]

We construct the adaptive controller

\[
U(t) = \begin{bmatrix} U_1(t) \\ U_2(t) \\ U_3(t) \end{bmatrix} = -\Gamma e + \begin{bmatrix} (e_1 - e_2)\alpha_1(t) \\ uw - xz + e_1\alpha_1 - (e_2 + e_3)\alpha_2 \\ xy - uv + e_3\alpha_3 \end{bmatrix}
\]

where

\[
\Gamma = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}
\]

is positive definite symmetric matrix, and the adaptive law

\[
\begin{align*}
\frac{d\alpha_1}{dt} &= (x-y)e_1 + xe_2 \\
\frac{d\alpha_2}{dt} &= -(x+y)e_2 \\
\frac{d\alpha_3}{dt} &= ze_3
\end{align*}
\]

In this simulation, let the parameters of the drive system be \(\theta_1 = 35\), \(\theta_2 = 28\), \(\theta_3 = 3\), the initial states of the drive and the controlled system are therefore \((1.8, 2, 0.6)^T\) and \((-0.8, -4, -3)^T\), respectively, and \((\alpha_1(0), \alpha_2(0), \alpha_3(0))^T = (0, 0, 0)^T\). The results of parameters identification and synchronization are shown in Fig.1 and Fig.2.

### 4 Simulation with Rössler Chaotic System

Consider the Rössler chaotic system in the following form:

\[
\begin{align*}
\frac{dx}{dt} &= -(y+z) \\
\frac{dy}{dt} &= x + \theta_1y \\
\frac{dz}{dt} &= \theta_2 + z(x-\theta_3)
\end{align*}
\]

which is chaotic when \(\theta_1 = \theta_2 = 0.2\), \(\theta_3 = 5\). In this case, we have

\[
\begin{align*}
\frac{du}{dt} &= -(v+w) + U_1 \\
\frac{dv}{dt} &= u + \alpha_1v + U_2 \\
\frac{dw}{dt} &= \alpha_2 + w(u-\alpha_3) + U_3
\end{align*}
\]

We construct the adaptive controller

\[
U(t) = \begin{bmatrix} U_1(t) \\ U_2(t) \\ U_3(t) \end{bmatrix} = -\Gamma e + \begin{bmatrix} e_1 \\ x-u-e_2\alpha_1 \\ xe_3 \end{bmatrix}
\]

where
is positive definite symmetric matrix, and the adaptive law

\[
\begin{bmatrix}
\frac{d\alpha_1}{dt} \\
\frac{d\alpha_2}{dt} \\
\frac{d\alpha_3}{dt}
\end{bmatrix} =
\begin{bmatrix}
ye_2 \\
-e_3 \\
z\alpha_3
\end{bmatrix}
\]

In this simulation, let the parameters of the drive system be \( \theta_1 = 0.2 \), \( \theta_2 = 0.2 \), \( \theta_3 = 5 \), the initial states of the drive and the controlled system are therefore \( (0.8, -2, 0.3)^T \) and \( (2, -0.4, 1.3)^T \), respectively. \((\alpha_1(0), \alpha_2(0), \alpha_3(0))^T = (0, 0, 0)^T \). The results of parameters identification and synchronization are shown in Fig.3 and Fig.4.

\[
\begin{array}{c}
\text{Fig.3 Synchronization error } e_1, e_2, e_3 \\
\text{Fig.4 Identification results of parameter } \theta_1, \theta_2, \theta_3,
\end{array}
\]

5 Conclusions

We have designed a novel adaptive controller for a class of chaotic systems dependent linearly on unknown parameters. The Chaos synchronization and parameters identification can be achieved simultaneously by the proposed controller. Using the analytic formula, it is easy and direct to construct the controller. Simulation results with Chen’s system and Rössler system show the effectiveness of the design method.

References


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