Analysis of Phased-Mission System Reliability and Importance with Imperfect Coverage

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Abstract  Accounting for static phased-mission systems (PMS) and imperfect coverage (IPC), generalized and integrated algorithm (GPMS-CPR) implemented a synthesis of several approaches into a single methodology whose advantages were in the low computational complexity, broad applicability, and easy implementation. The approach is extended into analysis of each phase in the whole mission. Based on Fussell-Vesely importance measure, a simple and efficient importance measure is presented to analyze component's importance of phased-mission systems considering imperfect coverage.

Key words  reliability; binary decision diagram for phased-mission systems; generalized and integrated algorithm; imperfect coverage model; fussell-veseley importance measure

A system that is subject to multiple, consecutive, and non-overlapping phases of operation is named a phased mission system (PMS). Many safety-critical systems encountered in the aerospace, nuclear, electronics, and other industries are actually PMS. The system configuration, success criteria and failure behavior may change with each phase. A classical example is an aircraft flight, which involves take-off, ascent, level flight, descent, and landing phases.

Because there are the s-dependence across the phases and the dynamic structure/ configuration, which usually requires a distinct model for each phase, reliability analysis of PMS is much more complex, compared to single-phase systems. Additionally, particularly in computer-based systems, there often exists imperfect coverage (IPC), which means an uncovered failure (also called single-point failure) can cause immediate system failure despite the presence of fault-tolerance mechanisms. Although uncovered failures can be addressed using IPCM, it further increases the complexity of analysis.

PMS in this paper is not repairable. Generally, there are two classes of models to address PMS problems: dynamic Markov-chain based models and static combinatorial models. Markov model is a powerful tool in reliability engineering. It can capture the dependencies, which exist between the basic event and the phases precisely. However, the size of the Markov model expands exponentially with the increase in the number of component, that is, the well-known state space explosion problem. Therefore, it is computationally intensive to solve the model. The common assumption of the Markov approaches is that the systems can be modeled using homogeneous, continuous time, and discrete state Markov chains. The combinatorial model is based on the assumption that the state of each component in the system has the characteristics of s-independence (Non s-dependence). The assumption has both advantages and disadvantages. On the one hand, it simplifies the analysis; on the other hand, it restricts the applicability of the model because s-dependence exists not only within a phase but also between phases. Even if s-dependence does not happen within a phase, it is likely to take place between phases. The PMS approach by Esary and Ziehms (EZ) is put forward in Ref.[1]. Component A, $C_a$, in phase $j$ is replaced by a set of s-independent mini-components $\{C_{ai}\}_{i=1}^{j}$ in series. This means that $C_a$ is operational in phase $j$ if and only if it functioned in all previous phases. Assume that $q_{ai}(t)$ is the failure probability at time $t$ on the condition that $C_a$ is normal at the beginning of phase $i$; while $F_{ai}(t)$ indicates the failure probability of $C_a$ at time $t$ during phase $j(0<t<T_j)$, then

$$F_{ai}(t) = \begin{cases} q_{ai}(t) & j=1 \\ 1 - \left[ \prod_{i=1}^{j-1} (1 - q_{ai}(T_i)) \right] (1 - q_{ai}(t)) & j>1 \end{cases} \tag{1}$$

This algorithm, however, will result in extremely enormous computation with the increase in the number of component.
of phases. What is more, it does not take the IPC into consideration. Ref.[2] uses phased-algebra to solve the dependence. Ref.[3] obtains the result of sum of disjoint products (SDP) through the combinatorial methods of phased-algebra and elimination. Nevertheless, the problem is that SDP will expand rapidly with the increase in the number of components and phases, as a result, the amount of data storage and computation will also expand dramatically. Considering the necessary work we have to do in finding out the minimum subset so as to calculate SDP, the amount of computation remains huge, although combinatorial model analysis is much less complicated than Markov-chain based models. The five-stepped simple and efficient algorithm (PMS-BDD) is introduced in Ref.[4]. On the basis of BDD, which is solved by Shannon decomposition, the final BDDs are obtained by combining each BDD of every phase by using phase-algebra. The approach also demonstrates the fact that the size of the BDD generated by backward phase-dependent operation (PDO) is smaller than that generated by forward PDO. The reason why BDD can be applied to PMS lies with the application of phase-algebra. PMS-BDD approach is of great value. Through the approach, the reliability of the system can be calculated as follows: work out the BDD of multi-phased system by using PMS-BDD, find out the regular SDP of all systems in BDDs, and then sum the probabilities together. The sum of all the probabilities stands for the reliability of the system. This approach, however, does not take IPC into consideration. Ref.[5] puts forward another approach, SEA, which is used to analyze single-phased mission system that contains IPC. SEA separates IPC analysis from the combinatorial model analysis and thus simplifies the analysis. The major contribution SEA has made is that, it allows reliability engineers the convenience to use their favorite software packet without considering the coverage concept; to figure out the corresponding reliability parameter, and then to get the final system reliability parameter, which is combined with the coverage, by slightly adjusting the results. Convenient as it is, SEA has its shortcomings. It only takes the single-phased mission system into consideration, while ignores PMS. Furthermore, because it assumes that components failure is statistically independent, it cannot explain the phenomenon that components failure is also related to dependence. Ref.[6] synthesizes those analytical approaches of combinatorial models, and puts forward generalized and integrated algorithm (GPMS-CPR) considering IPC. This approach is used to research PMS and IPC.

The first part of the article reviews the concepts related to GPMS-CPR, the second part discusses the improvement of GPMS-CPR algorithm; the third part analyzes importance measures, a conclusion is arrived at in the fourth part.

1 GPMS-CPR Concepts

1.1 Imperfect Coverage Modeling

Computer-based systems usually exhibit multiple failure modes: covered and uncovered failures. Additionally, different failure modes have diverse effects on the system failure.

“Covered failure” is local to the affected component, it might or might not lead to system failure which depends on the enduring redundancy.

“Uncovered failure” (or single-point failure) is globally malicious, causing system failure directly.

Fig.1 shows the general structure of a coverage model. The entry point to the model represents the occurrence of the fault, and the three exits denote three possible outcomes/events. If the fault is transient, and can be handled without discarding any component, then the transient restoration exit (labeled R) is taken. The permanent coverage exit (labeled C) signifies the determination of the permanent nature of the fault, the successful isolation and removal of the faulty component. If the permanent coverage exit is reached, a covered component failure is said to occur. When an uncovered fault (by itself) brings down the system and the uncovered failure exit (labeled S) is reached, an uncovered component failure occurs.

Within the context of reliability analysis it is required only to refer to the exit probabilities. The exits are mutually exclusive and complete: the three exit probabilities sum to 1. Therefore, \([r, c, s]\) is defined as
the probability of taking the transient restoration, permanent coverage, uncovered failure, respectively, given that a fault occurs[5,7]. More information about coverage modeling for fault tolerant computer-based systems is in Refs.[8,9].

1.2 GPMS-CPR Algorithm

To study CPR, IPC, and PMS, Ref.[6] puts forward a five-stepped GPMS-CPR algorithm. The algorithm synthesizes those effective approaches mentioned above so as to analyze the reliability of GPMS. As mentioned above, mini-components can deal with the dependence of components between phases. First change the system into equivalent mini-components system, and then apply SEA approach to PMS in the same way. Similar with SEA, Pr{no uncovered failure occurring}, $P_u$, is defined as

$$P_u = \prod_{A \in S} (1 - Pr(SF_A)) = \prod_{A \in S} (1 - u[A])$$

Let $SF_A$ signify that $C_A$ fails uncovered during the whole mission. Let $NF_{Aj}$ denote that $C_{Aj}$ doesn’t fail, $CF_{Aj}$ denote that $C_{Aj}$ fails covered, and $SF_{Aj}$ denote that $C_{Aj}$ fails uncovered. And then the probabilities of each of the three mutually exclusive events for a mini-component in the PMS are

$$n[A] = Pr(NF_{Aj}) = q_w u(A)$$
$$c[A] = Pr(CF_{Aj}) = c_w q_w u(A)$$
$$u[A] = Pr(SF_{Aj}) = s_w q_w u(A)$$

Let $NF_{Ai}$ signify that $C_i$ hasn’t failed before the end of phase $i$, $CF_{Ai}$ signify that $C_i$ has failed covered before the end of phase $i$, and $SF_{Ai}$ signify that $C_i$ has failed uncovered before the end of phase $i$, and then $u[A_i], c[A_i], n[A_i]$ are defined as follows

$$u[A_i] = Pr(SF_{Ai}) = u[A] + \sum_{j=1}^{i-1} \prod_{k=1}^{j} n[A_k] u[A_k]$$
$$c[A_i] = Pr(CF_{Ai}) = c[A] + \sum_{j=1}^{i-1} \prod_{k=1}^{j} n[A_k] c[A_k]$$
$$n[A_i] = Pr(NF_{Ai}) = \prod_{k=1}^{j} n[A_k]$$

when $j = 1$, then $u[A_i] = u[A], c[A_i] = c[A], n[A_i] = n[A]$

According to SEA, failure function $F_{Aj}(t)$ is the conditional failure probability of $C_A$ at phase $j$. That is, the probability of taking the transient restoration, permanent coverage, uncovered failure before the end of phase $j$.

$$F_{Aj}(t) = Pr(CF_{Aj} | SF_A) = \frac{c[A]}{u[A]} = \frac{c[A]}{u[A]}$$

The failure probability $U_{sys}$ at the end of the IPCM system’s mission can be calculated as follows: set up the static fault tree modeling of every phase in accordance with the demands and performance standards required by system phases; work out the PMS-BDD of all the phases by combining the single-phased BDD through phased-algebra; solve the failure probability $Q_j$ at the end of the PCM system’s mission with $F_{Aj}(t)$; and then combine it with $P_u$. It is similar to SEA. System unreliability is defined as

$$U_{sys} = P_u + QP_u$$

2 Improvements on GPMS-CPR Algorithm (IGPMS-CPR)

In GPMS-CPR algorithm, $P_u$ is defined as the probability under the condition that no uncovered failure happens to any components during the whole process of PMS. Nevertheless, this is not adequate enough to study the reliability of PMS at any given phase. In order to study the reliability of PMS at every phase, we define $P_{s_j}$ as the probability when no uncovered failure occurs to any components before the end of phase $j$. Let $s$ represent a set of all components in the system.

$$P_{s_j} = Pr\{no component experiences uncovered failure before the end of phase j\} = \prod_{A \in S} (1 - Pr(SF_A)) = \prod_{A \in S} (1 - u[A])$$

We assume that $U_{sys}$ is the failure probability of IPCM system at the end of phase $j$, $Q_j$ is the failure probability of PCM system when no uncovered failure happens to any components before the end of phase $j$. The “Total Probability Theorem” shows that the system unreliability and reliability can be evaluated as follows

$$U_{sys} = P_{s_j} + Q_j P_{s_j} \quad R_{sys} = P_{s_j} (1 - Q_j)$$

According to SEA and PMS-BDD, system reliability is the probability that any of SDP occurs, denoting that the system is operational. Assume $DPC_{ik}$ is one of the SDP which is generated by BDDs from the beginning of mission to phase $j$.

$$Pr\{DPC_{ik}\} = \prod_{A \in S} n(A) \prod_{B \in C} c(B) \prod_{C \in D} u(C) \quad i,k \leq j$$
Let \( o \) represent a set of the operational components, \( f \) denote a set of the covered failed components, \( t \) signify a set of all components except the components of \( o \cup f \), then

\[
\Pr \{ \text{DPCI}_j \} = \prod_{i \in o} \frac{n(A_i)}{u(A_i)} \prod_{j \in f} \frac{c(B_j)}{u(B_j)} \prod_{k \in t} \frac{1}{u(C_k)}
\]

Therefore

\[
R_{sys} = \sum \Pr \{ \text{DPCI}_j \} = \prod_{j \in o} \frac{n(A_j)}{u(A_j)} \prod_{j \in f} \frac{c(B_j)}{u(B_j)} = P_j \sum_{o} N(A_j) \prod_{j \in f} C(B_j) \tag{5}
\]

\[
N(A)_j = \frac{n(A_j)}{u(A_j)} \quad C(B_j) = \frac{c(B_j)}{u(B_j)} \tag{6}
\]

Obviously

\[
1 - Q_j = \sum_{j} N(A_j) \prod_{j \in f} C(B_j) \quad i, k \leq j \tag{7}
\]

When \( j = m \), Eq.(4) is simplified as Eq.(3) and \( C(B_j) \) in Eq.(6) is the same as \( F_j(t) \) in Eq.(2) of GPMS-CPR algorithm. Obviously, Eq.(4) covers Eq.(3), and it is more widely used and more practical. Eq.(3) in GPMS-CPR algorithm is, in fact, only a special case of Eq.(4).

3 Importance Measures

Fussell-Vesely importance measure means that a component that is included in several cut-sets has contribution to system failure even if it isn’t fatal. Based on the Fussell-Vesely ideas, the new simple and efficient importance measure considers both phased-mission systems and the imperfect coverage. Importance index of component \( C_a \) in phase \( j \) with respect to system unreliability during the whole mission, \( I_{j}^{\text{FVM}}(t) \), is defined as follows

\[
I_{j}^{\text{FVM}}(t) = \frac{g_{j}(t) + u(A_j)}{U_{qs}} \tag{8}
\]

Let \( g_{j}(t) \) signify the probability of the whole cut-sets including component \( C_a \) in phase \( j \). Operational and failure probability of each event of the cut-sets is calculated by the Eq.(6).

From the fault tree in Fig.2 in Ref.[6], we can get the whole cut-sets as listed in Tab.1. Based on the IGPM/CPR in terms of Tab.3, we obtain \( Q = 0.013866 \), \( U_{qs} = 0.014128 \), and then calculate \( I_{j}^{\text{FVM}}(t) \) according to Eq.(8), the results are listed in Tab.2.

![Fig.2 Equivalent system for end of mission](image-url)
As anticipated, components of the same type are equally important in each phase because they are functionally and statistically identical. The difference in importance across phases reflects the difference in requirements for each phase. From Tab.2, there are some interesting results which are different from that in Ref.[6] calculated by Birnbaum importance measure:

1) System designer should pay more attention to the group including event $D_a$, $D_b$, and $D_c$ in the phase 2, and then another group including event $C_a$ and $C_b$ in the phase 1 because these components contribute most to system unreliability or performance and should be improved. From the example system structure, it is obvious that events $D$ in the phase 2 and events $C$ in the phase 1 are more important intuitively. From the input parameters, it is clear that events $D$ in the phase 2 and events $C$ in the phase 1 are more important by instinct.

2) Importance values of the same component in different phases are different which isn’t direct relative to the increase of number of phases. The events $C$ in phase 1 are more sensitive than that in phase 3. The events $D$ in phase 2 are more sensitive than that in phase 3. But the events $B$ in phase 2 are more sensitive than that in phase 1. These results help the system designer identify which phases contribute most to component unreliability or performance and would be good candidates for upgrade.

These results shown by quantitative analysis are consistent with that by qualitative analysis of input parameters in Tab.3 and the structure of the example system[6].

<table>
<thead>
<tr>
<th>Tab.3</th>
<th>Input parameters for example system</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Phase 1 (33 h)</td>
</tr>
<tr>
<td>$P$</td>
<td>$P$ or $\lambda (10^{-6} \text{ h}) c$</td>
</tr>
<tr>
<td>$A_t$</td>
<td>0.000 1</td>
</tr>
<tr>
<td>$B_i$</td>
<td>$\lambda=1.50$</td>
</tr>
<tr>
<td>$C_i$</td>
<td>0.002 5</td>
</tr>
<tr>
<td>$D_i$</td>
<td>0.001 0</td>
</tr>
</tbody>
</table>

4 Conclusions

The static phased-mission system analysis approach is an important part of PMS reliability engineering. Improvements on GPMS-CPR for the static PMS and the imperfect coverage provides the theoretical support and simple and efficient algorithm for reliability engineers to analyze IPCM system. The simple and efficient importance measure is presented to find the weakness of the phased-mission system. The approaches extend the Fussell-Vesley importance measure into the static phased-mission system and IPC, and get a better result.

References


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