Exponential Stability for Delayed Cellular Neural Networks*  

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Abstract  The exponential stability of the delayed cellular neural networks (DCNN’s) is investigated. By dividing the network state variables into some parts according to the characters of the neural networks, some new sufficient conditions of exponential stability are derived via constructing a Liapunov function. It is shown that the conditions differ from previous ones. The new conditions, which are associated with some initial value, are represented by some blocks of the interconnection matrix.

Key words  delayed cellular neural networks;  exponential stability;  partitioned matrices

Cellular neural networks (CNN’s), introduced by Chua and Yang, have been extensively studied in the past decade[1]. It is significant to apply CNN’S to signal processing, especially in static image treatment. Although electronic circuits of CNN’s can be fabricated into chips by the large-scale integration technology, the finite switching speed of amplifiers and communication time will introduce the time delays in the interaction between neurons. Delayed cellular neural networks (DCNN’s), a generalization of the standard CNN’s model, were first introduced by Roska and Chua and then used in various types of motion-related applications such as processing of moving images, speed detection of moving objects and pattern classification[2,3,23]. So far, many stability criteria for DCNN’s have been obtained[4~10]. In Ref.[4], a sufficient condition for complete stability of DCNN’s with positive cell linking and dominant templates is given. In Ref.[5], it is proved that if sum of the feedback matrix and the delayed feedback matrix is symmetric and the length of delay is smaller than a certain value depending on the delayed feedback matrix, then the DCNN’s is completely stable.

In this paper, it is required that the delayed cellular neural network has an equilibrium point. We derive some conditions on exponential stability for DCNN’s, which are represented by some blocks of the interconnection matrix.

The dynamics of continuous time DCNN’s can be described by the following time-delayed functional differential equation

\[
\frac{dx}{dt} = -x(t) + Af(x(t)) + Bf(x(t - \tau)) + u \quad t \geq 0 \quad (1)
\]

where \(x(t)=[x_1(t), x_2(t), \cdots, x_n(t)]^T\) is the state vector, \(u=[u_1, u_2, \cdots, u_n]^T\) is constant vector, \(f(x)=[f_1(x_1), f_2(x_2), \cdots, f_n(x_n)]^T\) is the output, \(A=(a_{ij})_{n \times n}\) is the feedback matrix, \(B=(b_{ij})_{n \times n}\) is the delayed feedback matrix, \(x(t - \tau)=[x_1(t - \tau), x_2(t - \tau), \cdots, x_n(t - \tau)]^T\), \(\tau \geq 0\) is the delay parameter, and

\[
f_i(x_i) = \frac{1}{2}(|x_i| + 1 - |x_i| - 1) \quad i=1,2,\cdots,n
\]

The initial condition for a DCNN’s is given by

\[
x(t) = \phi(t) \quad t \in [-\tau^*, 0]
\]

where \(\phi(t)\) is assumed to be a continuous function on \([-\tau^*, 0], \tau^* = \max \tau_i\).

1 Preliminaries

Since \(|f_i(x_i(\cdot))| \leq 1\) is bounded, we have the following.

Lemma 1  There exists at least an equilibrium point for system (1).

We assume that \(x^*\) is the equilibrium point of system (1). By means of the transformation

\[
y(t) = x(t) - x^*
\]

then system (1) can be put into the equivalent system

\[
\frac{dy}{dt} = -y(t) + Af(y(t) + x^*) - f(x^*) + B(f(y(t - \tau) + x^*) - f(x^*)) \quad (2)
\]

To obtain our conditions, we will divide the set

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\(I = \{1, 2, \cdots, n\}\) as follows
\[
I = I_1 \cup I_2 \cup I_3
\]
where \(I_1 = \{i \in I | x_{i}^{*} > 1\}\), \(I_2 = \{i \in I | -1 \leq x_{i}^{*} \leq 1\}\), \(I_3 = \{i \in I | x_{i}^{*} < -1\}\).

We can rearrange the order of \(\{y_1, y_2, \cdots, y_n\}\), such that
\[
I_1 = \{1, 2, \cdots, r\} \quad I_2 = \{r+1, r+2, \cdots, r+m\} \\
I_3 = \{r+m+1, r+m+2, \cdots, n\}
\]
where \(r, m\) are non-negative integers. Variables of system (2) are reordered, but we use the same symbols as system (2).

Let
\[
y = (y_1^T, y_2^T, y_3^T)
\]
where \(y_{(1)} = (y_{11}, y_{12}, \cdots, y_{1n})^T\), \(y_{(2)} = (y_{21}, y_{22}, \cdots, y_{2n})^T\), \(y_{(3)} = (y_{31}, y_{32}, \cdots, y_{3n})^T\), so system (2) can be decomposed as
\[
\begin{align*}
\frac{dy_{(1)}}{dt} &= -y_{(1)} + A_{11} g(y_{(1)}) + A_{12} g(y_{(2)}) + A_{13} g(y_{(3)}) + B_{11} g(y_{(1)}(t-\tau)) + B_{12} g(y_{(2)}(t-\tau)) + B_{13} g(y_{(3)}(t-\tau)) \\
\frac{dy_{(2)}}{dt} &= -y_{(2)} + A_{21} g(y_{(1)}) + A_{22} g(y_{(2)}) + A_{23} g(y_{(3)}) + B_{21} g(y_{(1)}(t-\tau)) + B_{22} g(y_{(2)}(t-\tau)) + B_{23} g(y_{(3)}(t-\tau)) \\
\frac{dy_{(3)}}{dt} &= -y_{(3)} + A_{31} g(y_{(1)}) + A_{32} g(y_{(2)}) + A_{33} g(y_{(3)}) + B_{31} g(y_{(1)}(t-\tau)) + B_{32} g(y_{(2)}(t-\tau)) + B_{33} g(y_{(3)}(t-\tau))
\end{align*}
\]
where
\[
A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\
A_{21} & A_{22} & A_{23} \\
A_{31} & A_{32} & A_{33} \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\
B_{21} & B_{22} & B_{23} \\
B_{31} & B_{32} & B_{33} \end{bmatrix}
\]
\[
(g(y_{(1)}), g(y_{(2)}), g(y_{(3)})) = f(y(t) + x^*) - f(x^*)
\]
Let \(k = \min \{\min(x_{i}^{*} - 1), \min(1 - x_{i}^{*})\}\). From Eq.(3), we know that \(k > 0\). Assuming initial function
\[
|y_{(i)}(t)| < k
\]
due to the continuous dependence of the solution on initial value, \(T > 0\), for any \(t \in [-\tau, T]\), \(|y_{(i)}(t)| < k\). Therefore, \(\forall t \in [0, T]\), we have
\[
\begin{align*}
f(y_{(i)}(t) + x^*) - f(x^*) &= 0 \quad \forall i \in I_1 \cup I_2 \\
f(y_{(i)}(t-\tau) + x^*) - f(x^*) &= 0 \quad \forall i \in I_1 \cup I_3
\end{align*}
\]
thus
\[
g(y_{(i)}(t)) = g(y_{(i)}(t)) = 0 \\
g(y_{(i)}(t-\tau)) = g(y_{(i)}(t-\tau)) = 0
\]
For any \(t \in [0, T]\), it follows that
\[
\begin{align*}
\frac{dy_{(1)}}{dt} &= -y_{(1)} + A_{12} g(y_{(2)}) + B_{12} g(y_{(1)}(t-\tau)) \\
\frac{dy_{(2)}}{dt} &= -y_{(2)} + A_{22} g(y_{(2)}) + B_{22} g(y_{(2)}(t-\tau)) \\
\frac{dy_{(3)}}{dt} &= -y_{(3)} + A_{32} g(y_{(2)}) + B_{32} g(y_{(3)}(t-\tau))
\end{align*}
\]

\[\text{(5)}\]

\section{Main Results}
In this section, we consider the locally exponential stability for neural networks.

\textbf{Theorem 1} If
\[
\|A_{11}\| + 2\|A_{12}\| + \|A_{13}\| + \|B_{11}\| + 2\|B_{12}\| + \|B_{13}\| < 2
\]
holds, then system (1) is exponentially stable.

\textbf{Proof} By the condition of the theorem, there exists a \(\varepsilon > 0\), such that
\[
2 - \|A_{12}\| - 2\|A_{13}\| - \|A_{11}\| - \|B_{11}\| - \|B_{12}\| - \|B_{13}\| - \varepsilon > 0
\]
\[\text{(6)}\]

We construct the Liapunov function for system (5)
\[
V = \sum_{i=1}^{3} \int_{-\tau}^{t} y_{(i)}(s) e^{\varepsilon(t-s)} ds
\]
Along the trajectories defined by system (5), the derivative of \(V(y)\) is given as follows
\[
\frac{dV}{dt} \leq 2e^{\varepsilon T} \sum_{i=1}^{3} \|y_{(i)}(0)\|^2 + \|A_{11}\| \|y_{(1)}(0)\|^2 + \|A_{12}\| \|y_{(2)}(0)\|^2 + \|A_{13}\| \|y_{(3)}(0)\|^2 + \|B_{11}\| \|y_{(1)}(0)\|^2 + \|B_{12}\| \|y_{(2)}(0)\|^2 + \|B_{13}\| \|y_{(3)}(0)\|^2 \\
+ \varepsilon e^{\varepsilon T} \sum_{i=1}^{3} \|y_{(i)}(0)\|^2 \\
\leq -e^{\varepsilon T} (2 - \|A_{12}\| - \|A_{13}\| - \|A_{11}\| - \|B_{11}\| - \|B_{12}\| - \|B_{13}\| - \varepsilon) \|y_{(1)}(0)\|^2 \\
- e^{\varepsilon T} (2 - \|A_{12}\| - \|B_{12}\| - \varepsilon) \|y_{(2)}(0)\|^2 \\
- e^{\varepsilon T} (2 - \|A_{12}\| - \|B_{13}\| - \varepsilon) \|y_{(3)}(0)\|^2 \\
\leq 0
\]
Obviously, we have
\[
V(t) \leq V(0)
\]
thus
\[
V(t) \leq V(0)
\]
The proof of the theorem is completed.

References


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