Two Dimensional Spatial Independent Component Analysis and Its Application in fMRI Data Process*

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Abstract One important application of independent component analysis (ICA) is in image processing. A two dimensional (2-D) composite ICA algorithm framework for 2-D image independent component analysis (2-D ICA) is proposed. The 2-D nature of the algorithm provides it an advantage of circumventing the roundabout transforming procedures between two dimensional (2-D) image data and one-dimensional (1-D) signal. Moreover the combination of the Newton (fixed-point algorithm) and natural gradient algorithms in this composite algorithm increases its efficiency and robustness. The convincing results of a successful example in functional magnetic resonance imaging (fMRI) show the potential application of composite 2-D ICA in the brain activity detection.

Key words independent component analysis; image processing; composite 2-D ICA algorithm; functional magnetic resonance imaging

In recent years, blind source separation (BSS) by independent component analysis (ICA) has drawn much attention because of its potential applications in signal processing such as in speech recognition systems, telecommunication and medical signal processing\cite{1-3}. And one important application of ICA is in image processing\cite{4}, such as brain activity localization in the functional magnetic resonance imaging (fMRI) data\cite{5-7}. Currently, ICA copes with two dimensional (2-D) image data as a combination of one-dimensional (1-D) signal\cite{4}, i.e. the image data is firstly transformed column by column into 1-D signals and decomposed by an ICA algorithm for 1-D signals, then the decomposed signals were transformed back to 2-D images as the result\cite{5,6}. Apparently, this procedure is burdensome in calculation. In this paper, the ICA algorithm framework for 2-D image independent component analysis (2-D ICA) is proposed, which can avoid the roundabout transforming procedures in practice for image processing of two dimensional (2-D) image data.

One implementation of ICA is based on the infomax learning rule and it was implemented by natural gradient algorithm or fixed-point (Newton) algorithm\cite{1,8-14}. However, the natural gradient algorithm offers better stability, but there is need for convergence speed improvement; while Newton (fixed-point) algorithm facilitates a higher convergence speed, but its stability is subject to the selected initial values. In our recent fMRI data processing, we propose a composite ICA algorithm with better robustness and efficiency compared with natural gradient algorithm and fixed-point algorithm\cite{15}.

In this paper, based on the maximum likelihood contrast criterion, first the composite ICA algorithm approach was extended to 2-D for 2-D image processing\cite{15}. The effectiveness was further confirmed by simulation and a real fMRI data test.

1 Method

1.1 The 2-D Image Model

The 2-D image model can be expressed as

\[ x(\cdot;\cdot) = Ax(\cdot;\cdot) \]  

(1)

where \( A \) is a scalar matrix, \( A \in R^{m \times n} \), \( x(\cdot;\cdot) \) is the \( n \) observed image matrices, \( x(\cdot;\cdot) = [x_1(\cdot;\cdot), \ldots , x_n(\cdot;\cdot)]^T \), \( x(\cdot;\cdot) \in R^{m \times k} \), \( s(\cdot;\cdot) \) is the \( m \) original images matrices, i.e., \( s(\cdot;\cdot) \in R^{m \times k} \). In general, \( m < n \).

The goal of 2-D Image Independent Component Analysis (2-D ICA) is to find a linear transformation \( w \) for the observed images \( x(\cdot;\cdot) \) that makes the output \( y(\cdot;\cdot) = wx(\cdot;\cdot) \) as spatially independent as possible, where the \( m \) separated images \( y(\cdot;\cdot) \) are
derived from the \( n \) observed mixtures \( x(:,i) \) by the unmixing matrix \( w \) with elements \( w_{ij} \) \( (i = 1,2,\ldots,n, \ j = 1,2,\ldots,m) \).

We have
\[
y(:,i) = wx(:,i) = wA\hat{s}(:) = E\hat{s}(:) \tag{2}
\]
where \( y(:,i) \) is an estimate of the sources \( s(,:) \).

1.2 2-D ICA Principle

**Definition 1** The mutual information \( I(y(:,i)) \) of unmixing image \( y(:,i) \) is
\[
I(y(:,i)) = \int p(y(:,i)) \log \frac{p(y(:,i))}{\prod_{i=1}^{n} p(y_{i}(,)))} \, d\sigma \tag{3}
\]
The mutual information is always positive or equal to zero when the components are independent\(^{10}\), i.e. when \( p(y(:,i)) = \prod_{i=1}^{n} p(y_{i}(,))) \), \( I(y(:,i)) = 0 \). This fact means that 2-D ICA can be realized by minimizing the mutual information \( I^{10} \).

1.3 2-D Composite ICA Algorithm

The p.d.f of the observations \( x(:,i) \) can be expressed as\(^{10}\)
\[
p(x(:,i)) = \det(w) \, p(y(:,i)) \tag{4}
\]
where \( p(y(:,i)) = \prod_{i=1}^{n} p(y_{i}(,))) \) is the hypothesized distribution of \( p(s(,))) \). The log-likelihood of the p.d.f in Eq.(10) is
\[
L(y(:,i),w) = \log \, \det(w) + \sum_{i=1}^{n} \log p(y_{i}(,))) \tag{5}
\]
Minimizing the mutual information \( I(y(:,i)) \) is equivalent to maximizing the log-likelihood \( L(y(:,i),w) \) with respect to \( w \)\(^{10}\), a direct approach to maximize the log-likelihood is based on the natural gradient algorithm\(^{11}\)
\[
w^+ = w + \alpha D[\text{diag}(\beta) + E\{\phi(y(:,i))y(:,i)^{\top}\}]w \tag{6}
\]
where \( \beta = E\{y(:,i)\phi(y(:,i)))\), \( D = \text{diag}(1/\beta) - E\{\phi(y(:,i))\} \), and \( \alpha \) is a learning factor \( E\{\} \) denotes the mathematical expectation value of \( \{\}. \)

However, the fixed-point method strongly depends on the selection of the initialization point, and may lose the stability for such as in vivo fMRI data processing.

To avoid the drawbacks of the fixed-point iteration and natural gradient algorithms, we suggest using a composite algorithm for 2-D image processing as follows
\[
w^+ = w + \lambda[I - \phi(y(:,i))y(:,i)^{\top}]w + (1-\lambda)\alpha D[\text{diag}(\beta) + E\{\phi(y(:,i))y(:,i)^{\top}\}]w \tag{8}
\]

Apparently, it is an extending version of the original 1-D composite algorithm\(^{15}\). The parameter \( \lambda \) is the weight factor with the value between zero and one, when \( \lambda = 0 \), Eq.(8) reduces to the fixed-point algorithm\(^{12,13}\), and when the weight factor \( \lambda = 1 \), it changes to the natural gradient algorithm\(^{1,10,11}\). In the computing process, \( D \) is changing continually along with the iterative procedure, and the parameter \( \lambda \) may be regarded as an on-off factor. If \( D \) approaches infinity, the fixed-point will lose its stability. In this case, we choose parameter \( \lambda = 1 \), i.e. using the natural gradient method to avoid instability of the algorithm. When the parameter \( \lambda \) is specified with different values, we get a series of algorithms.

After we get unmixing matrix \( w \) with the composite-ICA algorithm, we calculate the independent image components \( y \) by \( y(:,i) = wx(:,i) \). Fig.1 shows a simulation example of images separation. Fig.1(a) illustrates the three original images and Fig.1(c) shows the separated signals from the mixed signals in Fig.1(b). Fig.1(c) shows that \( y \) is a permuted result of the original images.
2 Application in Localization of Brain Activities in fMRI Data

The fMRI data was collected at Beijing hospital. The stimulus was a red illuminant point presenting at the center of the visual field with frequency 8 Hz, light intensity 200 cd/m² and visual angle of 2°. Totally six transverse sections were collected in a bottom-up direction. Each section is composed of 128×128 voxels. Each section map was completely collected in 160 s resulting in 80 sample images alternating between stimulation and non-stimulation conditions.

Suppose that the distribution images of brain functional activities are independent of the background images, so the above composite 2-D ICA algorithm can be used to blindly separate the fMRI data into independent image components\(^5\sim7\). Using the composite 2-D ICA algorithm, we may obtain 80 separated component maps. Similar to the application of the 1-D ICA in fMRI\(^5\sim7\), we choose the first few separated component maps as the resultant maps, in which the voxels with their z-score values greater than a specified threshold, such as 6, are considered to be the active voxels. Then we calculate the correlation coefficients (CC) between the active voxels and the stimulative paradigm signal. The active voxels that CC values is larger than 0.65 is specified as the real active voxels, and it is set as black points in contrast to the background as fMRI imaging result map in Fig.2. The result show the physiological fact that the excited areas evoked by a visual stimulus are mainly in the region of the primary visual cortex: areas 17 and 18 of the occipital lobes\(^5\sim7\).

3 Conclusions

In this paper, the ICA algorithm framework for 2-D ICA is proposed, which can avoid roundabout transforming procedures in current ICA between two dimensional (2-D) image data and one-dimensional (1-D) signal. A composite 2-D ICA algorithm with a promising fusion of the stability and convergence is introduced here for 2-D signal component decomposition such as the image processing of an fMRI dataset. The experimental simulation results show that composite 2-D ICA is a potent tool. With the composite 2-D ICA algorithm, the brain activities in an fMRI dataset are successfully localized, which shows that composite 2-D ICA algorithm is of great potential in ICA blind sources separation and its application in fMRI.

References


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