Adaptive Trajectory Tracking Control of Wheeled Mobile Robots with Nonholonomic Constraint

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Abstract  A mobile robot is one of the well-known nonholonomic systems. In this paper, a new adaptive tracking controller for the kinematic model of a nonholonomic mobile robot with unknown parameters is proposed. Stability of the rule is proved through the use of a Liapunov function. The artificial electrostatic field cooperates with error posture in steering in the controller. At last, this method is implemented on the simulations and the wheeled mobile robot. Results show the effectiveness of the controllers.

Key words mobile robot;  nonholonomic systems;  trajectory tracking;  artificial electrostatic field

Wheeled mobile robots (WMR) have been widely applied in a lot of areas that range from the teleoperated Sojourner on the Mars Path finder mission to cleaning robots in the Paris Metro. As a result, the interest of the Robotics Community for the wheeled mobile robots’ tracking control problem has grown rapidly[1-13]. WMR is one of the well-known system with nonholonomic constrains and it contains a class of mechanical systems characterized by kinematic constrains that are not integrable and cannot therefore be eliminated from the model equations. From Brockett’s necessary conditions for stability, one may demonstrate that systems with nonintegrable velocity constrains cannot be stabilized to a point with smooth static-state feedback[1-2]. With this result, the control problems of the nonholonomic system become a challenge task. In the early researches, the controller for kinematic model was concentrated on[4-7]. The control input of the controller for kinematic model is generally velocity, but it is more realistic that the real input is torque. There are many researches on torque controller for the dynamic model in recent years[9-11]. However, these torque controllers can not show high performance in practical wheeled robot system because we can’t obtain all precise parameters of a wheeled mobile robot.

In this paper, a method is presented to design an adaptive tracking controller for the kinematic model of a nonholonomic mobile robot with unknown parameters. Stability of the rule is proved using Liapunov function. The artificial field cooperating with posture error is used to control the steering system of robotic, and the approach trajectory is improved. At last, these methods are implemented on the simulations and the autonomous mobile robot. The results of simulation and experiment prove the validity of the above methods.

1 Kinematics and Dynamics of a Nonholonomic Mobile Robot

Consider the following nonholonomic mobile robot that is subject to \( M \) constraints \(^3,11-12\)
\[
M(p) \frac{d^2 p}{dt^2} + V(p, \frac{dp}{dt}) \frac{dp}{dt} + G(p) = B(p) \tau + A^T(p) \lambda
\]  
(1)
where \( p \in R^n \) is generalized coordinates, \( \tau \in R^r \) is the input vector, \( \lambda \) is the vector of constraint forces, \( M(p) \in R^{n \times n} \) is a symmetric and positive-definite inertia matrix, \( V \in R^{n \times r} \) is the centripetal and coriolis matrix, \( G(p) \in R^{n} \) is the gravitational vector, \( B(p) \in R^{r \times n} \) is the input transformation matrix, and \( A(p) \in R^{n \times n} \) is the matrix associated with the constraints.

The Kinematic constraints are assumed to be expressed as
\[
A(p) \frac{dp}{dt} = 0
\]  
(2)

With respect to the dynamics of mobile robot Eq.(1), there is a parametric vector \( \eta \) on kinematic and dynamics which satisfies
\[
\frac{dp}{dt} = S(p)v(t)
\]

\[
\overline{M}(p)\frac{dv}{dt} + \overline{V}(p, \frac{dp}{dt})v + \overline{G}(p) = Y(p, \frac{dp}{dt}, v, \frac{dv}{dt})\eta
\]

where \( S(p) \in \mathbb{R}^{n \times (n-m)} \) spans the null space of \( A(p) \) and a full-rank matrix formed by a set of smooth and linearly independent vector fields, \( v \in \mathbb{R}^{n-m} \), \( \overline{M} = S^TMS \)

\[
\overline{V} = S^T(\overline{M} \frac{dS}{dt} + VS)
\]

\[
\overline{G} = S^T \overline{G}
\]

\( \overline{B} = S^T B \), \( Y \) is the matrix whose elements consists of known functions, and \( \eta \) is a parametric vector which is composed of the known and unknown elements\(^{[11]} \). Clearly, it is impossible to design a good tracking controller for dynamic model with these unknown parameters.

2 Adaptive Tracking Control of a Nonholonomic Mobile Robot

2.1 Error Posture and an Artificial Electrostatic Field Steering

There is a mobile robot which is located on 2D plane in which a global Cartesian coordinate system is defined. The robot in the world possesses three degree of freedom in its positioning as shown in Fig.1, and it is represented by a posture as follows

\[
p = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix}
\]

where \( \theta \) is the posture angle, \( x \) and \( y \) are the coordinates.

An electrostatic artificial field will be used to navigate the wheeled mobile robot. Fig.3 gives out a dummy artificial electrostatic field in global Cartesian coordinate system\(^{[12]} \). In our controller, we consider the reference posture as a new coordinate. The point \( R \) is the new grid origin and the \( x \)-aixs is in the direction of \( \theta \). So the artificial electrostatic field from reference posture to current posture as wire splice is shown in Fig.2: C-S-R.

\[
\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{d\theta}{dt} \end{pmatrix} = J\begin{pmatrix} v \\ \omega \end{pmatrix} = \begin{pmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} v \\ \omega \end{pmatrix}
\]

The Kinematic constraints are assumed to be expressed as

\[
\frac{dx}{dt}\sin \theta - \frac{dv}{dt}\cos \theta = 0
\]

In the control system, two postures are used, the reference posture \( p_r = (x_r, y_r, \theta_r) \), and current posture \( p_c = (x_c, y_c, \theta_c) \). A reference posture is the goal posture of the robot (point \( R \) in the Fig.2) and a current posture is its real posture (point \( C \) in the Fig.2). Defined error posture as \( p_e \)

\[
p_e = \begin{pmatrix} x_e \\ y_e \\ \theta_e \end{pmatrix} = \begin{pmatrix} \cos \theta_e & \sin \theta_e & 0 \\ -\sin \theta_e & \cos \theta_e & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} p_r - p_c \end{pmatrix}
\]

Fig.1 Mobile Robot with two actuated wheels

Fig.2 Error posture and artificial electrostatic field
2.2 Trajectory Tracking Control Problem

The architecture of a tracking control system for the robot is shown in Fig. 4. The global input of the system is the reference posture \( p_r \) and reference velocities \( q_r = (v_r, \omega_r)^T \), which are variables of time. The global output of the system is the current posture \( p_c \). The purpose of this tracking controller is to make the error posture converge at 0. Let us describe each component in Fig. 4 from left to right. The first box is error box. It is a calculation of an error from \( p_r \) and \( p_c \) using Eq. (7). The next box in Fig. 4 is a control rule for wheeled robot, which calculates target velocities \( \omega = \begin{pmatrix} v \cr \omega \end{pmatrix} \). The dotted box stands for the wheeled robot hardware capability of transforming target velocities to real current velocities. The next box is the kinematics Matrix \( J \) to produce the derivative of the current posture \( p_c \). The last box is for integration.

In Fig. 2, \( \beta \) is the angle between current posture direction and artificial electrostatic field. \( \partial \) is the tangential angle. Then the following equation is known from Ref. [12]:

\[
\begin{align*}
\theta_e + 2\partial &= \beta \\
\theta_e + \partial &= \arctg(y_e/x_e)
\end{align*}
\]

Then
\[
\beta = 2\arctg(y_e/x_e) - \theta_e
\]

2.3 A Control Scheme and Its Stability

In this section, we will find a stable control rule using a Liapunov function. The following lemma follows the systems depicted in Fig. 1.

Using Eqs. (5)-(7), the following equation holds

\[
\begin{bmatrix}
\frac{dx_e}{dt} \\
\frac{dy_e}{dt} \\
\frac{d\theta_e}{dt}
\end{bmatrix} = \begin{bmatrix}
y_e \omega_e - v_e + v_e \cos \theta_e \\
x_e \omega_e + x_e \sin \theta_e \\
\omega_e - \omega_e
\end{bmatrix}
\]

Define \( q_e \) as

\[
q_e = \begin{pmatrix} v_e \\
\omega_e \end{pmatrix} = \begin{pmatrix} v_e - v \\
\omega_e - \omega \end{pmatrix}
\]

then

\[
q_e = \begin{pmatrix} v_e \\
\omega_e \end{pmatrix} = \begin{pmatrix} v + v_e \\
\omega + \omega_e \end{pmatrix}
\]

Propose a specific instance of the control rule for the target velocities as follows
where $K_x$, $K_y$, $K_\theta$, $K_\beta$ are positive constants.

Propose a scalar function $V$ as a Liapunov function candidate:

$$V = \frac{x^2}{2} + \frac{y^2}{2} + \frac{1 - \cos \theta}{K_y} + \frac{v^2}{2} + \frac{\omega^2}{2}$$

then

$$\frac{dV}{dt} = \frac{dv}{dt} x + \frac{dy}{dt} y + \frac{d\theta}{dt} \frac{\sin \theta}{K_y} + \frac{dv}{dt} v + \frac{d\omega}{dt} \omega = -v v + x v \cos \theta + y v \cos \theta + \sin \theta \frac{\omega - \omega}{K_y} + \frac{dv}{dt} v + \frac{d\omega}{dt} \omega = -k x^2 - v K_\beta (x^2 + y^2) \sin \beta - y \omega \frac{\sin \theta}{K_y} + \frac{dv}{dt} v + \frac{d\omega}{dt} \omega$$

Now, the parameter update rules are chosen as:

$$\frac{dv}{dt} = \frac{v K_\beta (x^2 + y^2) \sin \beta}{K_y v} + \frac{x - v}{x - v}$$

$$\frac{d\omega}{dt} = \frac{\sin \theta}{K_y - \omega}$$

then

$$\frac{dV}{dt} = -k x^2 - v K_\beta \sin^2 \beta - v \omega^2 - v \omega^2 \leq 0$$

Soundness of Eqs.(12)~(14) are established by this Liapunov function.

3 Simulation Results

In this section, we perform computer simulation on the kinematics model of a mobile robot by using the designed adaptive tracking controller. In this simulation, physical parameters of the wheeled robot are chosen from real FIFA MIROSOT robot system.

The mobile robot is asked to track the forward spinning trajectory. In this tracking, we assume that there is a linear error between $q_c$ and $q$. The adaptive controller must overcome this error. Fig.5 shows the track result. We can see the error posture between real robot and reference robot in Fig.6.

4 Control Parameters

In the previous section, we demonstrated that the system is stable for any combination of parameter
value of $K_x$, $K_y$, $K_\theta$ and $K_\beta$. However, since we need a non-oscillatory, but not too slow response of a robot, we have to find an optimal parameter set. In order to simplify the analysis, we consider only situations in which reference posture is moving to the positive direction at a constant velocity. The velocities of the reference robot are selected as follows

$$v_r = 0.125 \text{ m/s}, \quad \omega_r = 0$$

It is very difficult to confirm an optimal parameter set because a suitable adjustment must be done among so many control parameters. Here, we try to find a better parameter set using intelligence genetic algorithm. Propose a function $Q$ as an evaluating indicator.

$$Q = \frac{1}{N} \sum_{i=1}^{N} \left[ \frac{\beta_i}{\pi} + \frac{\hat{S}_i}{S_0} \right]$$

(15)

where $i=1,2,\cdots,N$ corresponds to the number of elapsed time step, $\beta_i$ is the $\beta$ in Fig.2, $\hat{S}_i$ is the maximum distance between real robot and reference robot, $S_0$ is the distance between real robot and reference robot.

At the beginning of the genetic algorithm evolution, we select the first parameter set. $K_x = 2.582353$, $K_y = 4.623529$, $K_\theta = 7.094118$, $K_\beta = 1.658824$. Fig.7 shows the simulation results with these parameters. At the end of the genetic algorithm evolution, we select the second parameter set. $K_x = 2.054902$, $K_y = 14.901256$, $K_\theta = 8.027451$, $K_\beta = 13.182353$. Fig.8 shows the simulation results with these parameters. We can see the error posture between real robot and reference robot in Fig.9. The meaning of $A$ is the error of the posture angle and the $R$ is the error of the position in Fig.9.

From these simulation results, we confirm that we can find better control posture through genetic algorithm. In the following experiments and in our real implementation, we use the second control parameter set.

## 5 Implementation

In this section, we will describe how the theory was implemented on the FIFA MIROSOT.

At first, robot is required to track the circle trajectory. The circle is shown in Fig.10. The velocities of the reference robot are selected as follows

$$v_r = 0.1693 \text{ m/s}, \quad \omega_r = -0.5 \text{ rad/s}$$

Second, robot is required to track the line trajectory. The line is shown in Fig.11. The velocities of the reference robot are selected as follows

$$v_r = 0.20 \text{ m/s}, \quad \omega_r = 0 \text{ rad/s}$$
Fig.10  The robot is tracking the circle trajectory

Fig.11  The robot is tracking the line trajectory

We can see the error posture between real robot and reference robot in Fig.12.

![Image](image_url)

(a) Track the circle trajectory

(b) Track the line trajectory

Fig.12  The error posture versus time (MIROSOT)

6 Discussions and Conclusion

For our tracking controller system, reference paths designated by $p_r$ and $q_r$ should satisfy the following conditions for “smoothness”: 1) the path itself is continuous; 2) the path has tangent direction continuity; 3) the path curvature is continuous; 4) the derivative of $\frac{d\omega_r}{dt}$ is bounded. Hence, the derivative of the curvature is also bounded. These conditions are a continuity requirement for robot path planning. However, if the non-smoothness paths exist, the target velocities will be too large to be attained by a real wheeled robot, and the linear or rotational acceleration might become too large causing the robot’s slippage. Therefore, we should give a limit to velocities and accelerations in the practical application.

In practical application, we cannot obtain all precise parameters of a wheeled mobile robot. As a result, we cannot give the wheeled robot a precise control quantity, and the robot cannot show high performance if we ignore this error. In this paper, we proposed a designed method of an adaptive controller to deal with this problem. A new adaptive tracking controller with a state derivative feedback procedure is presented based on the kinematic models. In this controller, an artificial electrostatic field is used to navigate the wheeled robot. The robustness of the controller is confirmed by Lyapunov stability theory, and the effectiveness of the controller is demonstrated via computer simulation and their implementation.

References


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