Controller Design for a Teleoperation System with Time Delay

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Abstract A robust controller design method is presented to guarantee the stability and zero tracking error for teleoperation system with time delay. Through choosing appropriate master and slave parameters, extended state equation about master and master-slave error is achieved, which can be analyzed by using time delay knowledge. Thus delay-independent and delay-dependent criteria are derived in terms of the Lyapunov stability theorem, control parameters are obtained by the feasible of linear matrix inequalities. Experimental results show the validity of these approaches and the performance of master and slave manipulators with delay variations is analyzed.

Key words teleoperation; time delay; delay-independent and delay-dependent criteria

Teleoperation systems are used to perform remote tasks by master and slave manipulator. When the slave manipulator contacts the environment, the interaction force is reflected to the operator to improve the performance of the teleoperated tasks. If this force is directly provided to the operator via the master, the teleoperation system is bilaterally controlled\[1\]. Time delay can appear in the information transmission between the master and the slave if a huge distance exists or a slow communication channel is used. This time delay affects negatively the system stability. Lots of bilateral teleoperation systems have been proposed to overcome the time delay problem\[2-3\]. For example, Lawrence used impedance models to describe the stability and transparency of various teleoperation schemes accounting for transmission delays\[4\]. Leung, Francis, and Apkarian designed a controller based on $\mu$-synthesis, while Kim, Hannaford, and Bejczy proposed and investigated shared compliant control\[5\]. Eusebi and Melchiorri further introduced criteria for stability, both independent and dependent on time delay\[6\]. Prediction, generally in the form of Smith predictors can be combined with wave-based systems to reduce the effects of the delay\[7\]. Considerable attention has been devoted to Internet-based teleoperation, in which the communications delay is variable. They have been further extended to including estimation of the delay, prediction of the delay, as well as Smith predictors in combination with energy regulation\[8-10\]. Niemeyer surveyed and explored the development of the wave variable concept applied to time-delayed teleoperation, under the assumption of an unknown but constant delay, by using impedance matched design, position feedback, and optional wave filtering, a system with consistent and predictable behavior was constructed\[11\]. It becomes transparent to the user if the delay remains below the human reaction time.

However, most of these literatures use a passivity-based formalism to construct teleoperation systems and the conservative selection of dissipating element applied to maintain system stability leads system to imperfect operation. Azorin J. M, et al. presented a generalized design, control method and the dynamic analysis for teleoperation systems with communication time delay based on the state formulation, but they used the Taylor expansion of the first order, and if a Taylor expansion of bigger order is applied, it is impossible to obtain all the necessary design equations\[12-13\].

In this paper, we employ the equal transformation to design the controller, which is based on delay-independent and delay-dependent analysis. The controller parameters can be obtained via the feasibility of the LMIs. With these controllers, not only the stability of the master and slave manipulators is guaranteed but also the master is tracked well by the slave finally, which is illustrated by the simulation results.

1 Modeling of the Teleoperation System with Time Delay

We only consider one dof for the master and slave in the teleoperation system. The simple linear model of an element with one dof is:

$$\frac{d^2 \theta}{dt^2} + b \frac{d\theta}{dt} = u(t)$$  (1)
where $J$ is the inertia of the element, $\frac{d\theta}{dt}$ is the rotate angle velocity, $b$ is the viscous fiction coefficient and $u(t)$ is the control torque applied. Convert it into the state space formulation, we have:

$$\begin{bmatrix} \frac{dx_m}{dt} \\ \frac{dx_s}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_m(t) \\ x_s(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_m(t)$$

(2)

$$y_m(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_m(t) \\ x_s(t) \end{bmatrix}$$

(3)

Considering all the possible interactions that could appear in the operator-master-slave-environment set, the model of the teleoperation system with time delay is shown in Fig.1.

Fig.1 The structure of the teleoperation system

where $F_m$ represents the force that the operator applies to the master, $u_m$ and $u_s$ are the master and slave control signals, $x_m$ and $x_s$ are the master and slave state vectors, $y_m$ and $y_s$ are the master and slave outputs. $G_2$ is influence on the slave of the force that the operator applies to the master. $k_e$ is feedback matrix of the master state. $k_i$ is feedback matrix of the slave state. It allows considering the interaction force of the slave with the environment, $R_m$ is interaction slave-master. It allows modeling the force reflection to the master. $R_s$ is interaction master-slave.

To consider force feedback from the slave to the master, the structure of the matrix $R_s$ must be:

$$R_s = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} = \begin{bmatrix} k_f & k_c \\ k_b & \end{bmatrix}$$

where $k_f$ is the force feedback gain. $k_c$ and $b$ are respectively the stiffness and the viscous friction of the environment\[12\]. So the control parameters that are necessary to be obtained are: $k_m$, $k_s$, $R_s$, $G_2$.

From the model shown in Fig.1, the master control signal $u_m(t)$ and the slave control signal $u_s(t)$ could be observed, respectively:

$$u_m(t) = k_m x_m(t) + R_m x_s(t-T) + F_m(t)$$

(6)

$$u_s(t) = k_s x_s(t) + R_s x_m(t-T) + F_s(t-T)$$

(7)

Replacing the master and slave control signal Eqs.(6), (7) in the master and slave state equation, the next state equations are obtained:

$$\frac{dx_m}{dt} = (A_m + B_m k_m) x_m(t) + B_m R_m x_s(t-T) + B_m F_m$$

(8)

$$\frac{dx_s}{dt} = (A_s + B_s k_s) x_s(t) + B_s R_s x_m(t-T) + B_s G_2 F_s(t-T)$$

(9)

2 Design of Controller

We give the following lemma firstly, that is necessary to the proof.

Lemma 1 For any real vector $D$ and $E$ with appropriate dimension and any positive scalar, we have:

$$DE + E^T D^T \leq \varepsilon DD^T + \varepsilon^{-1} E^T E$$

Select appropriate value to make following condition satisfied:

$$A_m + B_m k_m = A_s + B_s k_s , \quad B_s R_s = B_s R_s , \quad B_s = B_s G_2$$

Assuming that the operator force $F_m$ is constant, we can obtain the following relation:

$$\frac{k_m}{J_m} = \frac{k_m}{J_s} , \quad \frac{k_m - b_m}{J_m} = \frac{k_s - b_s}{J_s}$$

$$\frac{R_m}{J_m} = \frac{R_m}{J_s} , \quad \frac{R_m}{J_m} = \frac{R_s}{J_s} , \quad G_2 = \frac{J_m}{J_s}$$

Then, the next state equation is yielded:
\[
\begin{align*}
\frac{dx_m}{dt} &= \begin{bmatrix} A_m + B_m k_m & 0 \\ 0 & A_m + B_m k_m \end{bmatrix} \begin{bmatrix} x_m \\ e \end{bmatrix} + \\
&+ \begin{bmatrix} B_m R_m & -B_m R_m \\ 0 & -B_m R_m \end{bmatrix} \begin{bmatrix} x_m(t-T) \\ e(t-T) \end{bmatrix}
\end{align*}
\]

(10)

Obviously, Eq.(10) is obtained by using the suited parameters. Based on this state equation, we can use the knowledge of time delay to analyze the error and the stability of the master-slave system, and give the design of controller. Without Taylor expansion, this transfer method can avoid the error of approximate the time delay and is considered as equivalence [12-13]. State equation eliminates coupling of the master and slave state and is converted into a simple time delay system, thus it’s easy to give the following results including delay-independent and delay-dependent.

### 2.1 Delay-Independent Analysis

We use delay-independent method to analyze system Eq.(10), and achieve the following theorem.

**Theorem 1** Given a system Eq.(10) with time delay, the error converges to zero, \( x(s) \to x(m) \) and the system is stable, if there exist \( P > 0 \) and \( \varepsilon_i > 0 \) satisfying the following LMI:

\[
\begin{bmatrix}
A_1 & \varepsilon_i B_m R_m R_m^{-1} B_m^T x \\
A_1 & 0 \\
* & -\varepsilon_i I
\end{bmatrix} < 0
\]

(11)

where \( x = P^{-1}, w = k_m x \)

\( A_1 = (A_m x + B_m w) + (A_m x + B_m w)^T + 2\varepsilon_i B_m R_m R_m^{-1} B_m^T \)

**Proof** Define the following Lyapunov function candidate for system Eq.(10). For the simpleness of evolve, we define the following expression.

\[
V = \xi^T L_\xi + \int_{t-T}^t \varepsilon_i^{-1} \xi^T (t) \xi(t) dt
\]

where

\[
\begin{bmatrix}
x_m \\
e
\end{bmatrix} = \xi(t), \quad B = \begin{bmatrix} B_m R_m \\ 0 \end{bmatrix}, \quad L = \begin{bmatrix} P \\ 0 \end{bmatrix}
\]

and \( P = P^T > 0 \). By using Lemma1, we have:

\[
\begin{align*}
\frac{dV}{dt} &= 2\xi^T L_\xi \dot{\xi} + 2\xi^T L_\xi \dot{\xi}(t-T) + \\
&+ \varepsilon_i^{-1} \xi^T (t) \xi(t) - \varepsilon_i^{-1} \xi^T (t-T) \xi(t-T)
\end{align*}
\]

\[
\begin{align*}
\frac{dV}{dt} &\leq 2\xi^T L_\xi \dot{\xi} + \varepsilon_i^{-1} \xi^T (t-T) \xi(t-T) + \\
&+ \varepsilon_i \xi^T LBB^T L_\xi + \varepsilon_i^{-1} \xi^T \xi - \varepsilon_i^{-1} \xi^T (t-T) \xi(t-T)
\end{align*}
\]

Using schur complements, we can achieve:

\[
\begin{bmatrix}
LA + A^T L + \varepsilon_i LBB^T L \\ I
\end{bmatrix} < 0
\]

(12)

then substitute \( A, B \) and \( L \) for the initial form, we obtain Eq.(11), the proof is completed.

The result of time-independent is conservative. If this condition is satisfied, the system is stable in spite of time delay. But this condition is so strong, especially, to some small time delay systems. Therefore, we discuss the delay-dependent analysis next.

### 2.2 Delay-Dependent Analysis

Usually, the time delay can be measured by repeating experimentation, so we suppose the time delay is known and use delay-dependent method to analyze the stability of system Eq.(10) and give the following theorem.

**Theorem 2** Given a system Eq.(10) with time delay \( T \) is a constant value, the error converges to zero, namely \( x(s) \to x(m) \) and the system is stable, if \( P > 0 \) and \( \varepsilon_i > 0 \), satisfy the following LMI.

\[
\begin{bmatrix}
A_1 & -B_m R_m x \\
A_2 & -A_2 \\
* & -\varepsilon_i I
\end{bmatrix} < 0
\]

(13)

where
\[
A_{11} = A_n x + B_n w + (A_n x + B_n w)^T + 2B_n R_n x
\]
\[
A_{12} = -T \varepsilon_1 (A_n x + B_n w) B_n R_n
\]
\[
A_{13} = -T \varepsilon_1 (A_n x + B_n w)^T
\]
\[
A_{14} = T \varepsilon_1 (A_n x + B_n w)^T
\]
\[
x = P^T, \quad w = k_n x, \quad v = T \varepsilon_1
\]

**Proof** Using the expression similar with the above delay-independent analysis. Choose the following Lyapunov function for the system Eq.(10)

\[
V = \dot{\zeta}^T L \dot{\zeta} + \int_{-T}^{t} \int_{-T}^{t} \varepsilon_1 \frac{d \dot{\zeta}^T}{dt} \frac{d \zeta}{dt} dt d\beta
\]

for \( t \geq T \) where

\[
\dot{\zeta}(t-T) = \dot{\zeta}(t) - \int_{t-T}^{t} \frac{d \dot{\zeta}}{dt} dt
\]

\[
\frac{d \dot{\zeta}}{dt} = A \dot{\zeta}(t) + B \zeta(t) - B \int_{t-T}^{t} \frac{d \dot{\zeta}}{dt} dt
\]

By using Lemma 1 we have:

\[
\frac{dV}{dt} = 2 \dot{\zeta}^T L \dot{\zeta}(t) - \varepsilon_1 \int_{t-T}^{t} \frac{d \dot{\zeta}^T}{dt} \frac{d \zeta}{dt} dt +
\]

\[
2 \dot{\zeta}^T \dot{L} \zeta(t) - 2 \dot{\zeta}^T L \dot{\zeta} + \varepsilon_1 \frac{d \zeta}{dt} \frac{d \zeta}{dt} dt + T \varepsilon_1 \frac{d \dot{\zeta}^T}{dt} \frac{d \zeta}{dt} dt 
\]

\[
= 2 \dot{\zeta}^T L \dot{\zeta} + 2 \dot{\zeta}^T \dot{L} \zeta + \varepsilon_1 \frac{d \dot{\zeta}^T}{dt} \frac{d \zeta}{dt} dt + T \varepsilon_1 \frac{d \dot{\zeta}^T}{dt} \frac{d \zeta}{dt} dt
\]

\[
= 2 \dot{\zeta}^T L \dot{\zeta} + \varepsilon_1 \frac{d \dot{\zeta}^T}{dt} \frac{d \zeta}{dt} dt + T \varepsilon_1 \frac{d \dot{\zeta}^T}{dt} \frac{d \zeta}{dt} dt
\]

using Schur’s complements, we can obtain Eq.(14)

\[
\begin{bmatrix}
LA + A^T L + 2LB & vA^T & LB & vA^T \\
* & B^T B & 0 & 0 \\
* & * & -\varepsilon_1 I & 0 \\
* & * & * & -vI
\end{bmatrix} < 0
\]

then substitute \( A, B \) and \( L \) for the initial form, we obtain Eq.(13), the proof is complete.

According to the known \( T \), we obtain the different controller parameters. Comparing with the time-independent results, it can reduce the conservation.

### 3 Simulation Example

To perform the delay-independent and delay-dependent analysis of the teleoperation system, we consider the next parameters: 1) Master: \( J_a = 1.5 \text{ kgm}^2 \), \( B_a = 11 \text{ Nm/ rad} \cdot \text{s}^{-1} \); 2) Slave: \( J_s = 2 \text{ kgm}^2 \), \( B_s = 15 \text{ Nm/ rad} \cdot \text{s}^{-1} \); 3) Environment: \( k_e = 100 \text{ Nm} \cdot \text{rad}^{-1} \), \( b_e = 1 \text{ Nm/ rad} \cdot \text{s}^{-1} \); 4) Force feedback gain: \( k_f = 0.1 \); 5) The operator exerts a constant force: \( F_a = 1 \text{ Nm} \); 6) The initial states of master and slave are: \( x_n(t) = 0.5 \text{ rad} \), \( x_s(t) = -0.2 \text{ rad} \).

#### 3.1 Simulation of Delay-Independent Analysis

The control parameters have been calculated solving the LMI Eq.(11) through the Matlab LMI Toolbox. The control parameters obtained are:

\[
k_n = [-144.8358 \quad -10.9994]
\]

\[
k_s = [-193.211 \quad -14.347]
\]

\[
R_e = [13.334 \quad 0.1334], \quad G_e = 1.334
\]

Simulation results can be seen in Fig.2: master (solid lines), slave (dotted lines).

![Simulation Results](image.png)

**Fig.2** Master and slave position tracking under time-independent controller

#### 3.2 Simulation of Delay-Dependent Analysis

By delay-dependent analysis, the control
parameters are calculated by solving the LMI in Eq.(13) through the Matlab LMI Toolbox. The parameters are different with different time delay and the parameter values are smaller than time-independent results.

1) When \( t = 0.1 \) s,

\[
k_n = \begin{bmatrix} -38.0597 & -11.3696 \end{bmatrix}, \\
k_s = \begin{bmatrix} -50.7716 & -14.8410 \end{bmatrix}
\]

2) When \( t = 0.5 \) s,

\[
k_n = \begin{bmatrix} -38.4275 & -11.1285 \end{bmatrix}, \\
k_s = \begin{bmatrix} -51.2623 & -14.5190 \end{bmatrix}
\]

3) When \( t = 1 \) s,

\[
k_n = \begin{bmatrix} -38.1388 & -11.2624 \end{bmatrix}, \\
k_s = \begin{bmatrix} -50.8772 & -14.6980 \end{bmatrix}
\]

From Fig.2 and Fig.3, we can observe that, with delay increases, either delay-independent controller or delay-dependent controller can guarantee the slave follows the master finally. To the respond speed of the system, the delay-independent controller is faster than the delay-dependent one. But there exists some vibration between the master and slave position in the former and when the time delay is very large, the latter LMI is infeasible.

4 Conclusions

In this paper, a controller design method for teleoperation systems with communication time delay has been presented. This method is based on the state space formulation and it allows the master and the slave to reach the same stable position. In order to use delay-independent and delay-dependent analysis, we must select appropriate parameters made identical coefficient of master and slave state space formulation. The advantage of the design method resides to obtain the control parameters and to achieve that the slave follows the master in the case of unknown time delay and known time delay, only a LMI must be solved. It is unfortunate that the nonlinear and uncertainty contain in system model are not considered.

References

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