An MMSE Decoding Algorithm without Matrix Inversion in QSTBC*

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Abstract The matrix inversion operation is needed in the MMSE decoding algorithm of orthogonal space-time block coding (OSTBC) proposed by Papadias and Foschini. In this paper, an minimum mean square error (MMSE) decoding algorithm without matrix inversion is proposed, by which the computational complexity can be reduced directly but the decoding performance is not affected.

Key words quasi-orthogonal space-time block coding (QSTBC); multiple input multiple output (MIMO) channel; minimum mean square error (MMSE) decoding algorithm

In order to extend the idea of space-time block codes (STBC) for two antennas proposed by Alamouti to more antenna scenario, there are two ways to go [1]. One attractive approach is the orthogonal STBC based on the full diversity put forward by V. Tarokh [2-3]. Another effective method is the quasi-orthogonal STBC (QSTBC) based on the full transmission rate finished by H. Jafarkhani, which gets full transmission rate in the cost of reducing the orthogonality of the coding matrix [4-5].

For QSTBC, there are two effective decoding algorithms, maximum likelihood (ML) algorithm and minimum mean square error (MMSE) algorithm [5-6]. MMSE decoding scheme solves the decoding output in one dimension, therefore it is more effective than ML decoding scheme in computational burden. But the matrix inversion operation is needed. In this paper, an effective MMSE decoding algorithm without matrix inversion is proposed, which will reduce the computational complexity obviously, and not affect the decoding performance.

1 H. Jafarkhani QSTBC Model

By using the orthogonality of the transmitted symbols, Alamouti STBC transmission matrix can be written by

$$A_{12} = \begin{bmatrix} c_1 & c_2 \\ -c_2^* & c_1^* \end{bmatrix}$$ (1)

where the subscript 12 indicates the transmitted symbols $c_1$, $c_2$. Based on Alamouti STBC idea, H. Jafarkhani gave the QSTBC transmission matrix for four transmit antennas as

$$C_J = \begin{bmatrix} A_{12} & A_{14} \\ -A_{41} & A_{34} \end{bmatrix} = \begin{bmatrix} c_1 & c_2 & c_3 & c_4 \\ -c_2 & c_1 & -c_4 & c_3 \\ -c_3 & -c_4 & c_1 & c_2 \\ c_4 & -c_3 & -c_2 & c_1 \end{bmatrix}$$ (2)

where $A_{12}$, $A_{14}$ are Alamouti matrices.

The received signals $r_1, r_2, r_3, r_4$ of a single antenna at time slot $t = 1, 2, 3, 4$ can be expressed by

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} c_1 & c_2 & c_3 & c_4 \\ -c_2 & c_1 & -c_4 & c_3 \\ -c_3 & -c_4 & c_1 & c_2 \\ c_4 & -c_3 & -c_2 & c_1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}$$ (3)

where $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are the path gains, and $v_1, v_2, v_3, v_4$ are the additive noises [6]. Eq.(3) also can be written by

$$\begin{bmatrix} r_1 \\ r_2^* \\ r_3^* \\ r_4^* \end{bmatrix} = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ -\alpha_1^* & \alpha_3^* & -\alpha_2^* & \alpha_4^* \\ -\alpha_3 & \alpha_1 & -\alpha_4 & \alpha_2 \\ \alpha_4 & -\alpha_3 & -\alpha_2 & \alpha_1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2^* \\ v_3^* \\ v_4 \end{bmatrix}$$ (4)

or by

$$r = Hc + v$$ (5)

The sparse matrix can be obtained by

$$A = A^H = HH$$ (6)
where the elements $x$, $y$ are
\[ x = \sum_{i=1}^{24} a_i^2, \quad y = (a_i a_i^* + a_i^* a_i + a_i^* a_i) \] (7)

2 QSTBC MMSE Decoding Algorithm without Matrix Inversion

From Eq.(6), pre-multiplying by $H^H$, we can get
\[ \hat{r} = H^H r = H^H H c + H^H v = A c + \tilde{v} \] (8)

The decoding output can be written by $W_i^H \hat{r}$, and the weight matrix $W_i^H$ can be obtained by minimizing the following MMSE criterion
\[ \min_{W_i^H} ||W_i^H \hat{r} - d||^2 \] (9)

The Wiener solution for $W_i^H$ can be expressed by
\[ W_i^H = (\sigma_i^2 A^{-1}) (\sigma_i^2 A A^{-1} + \sigma_i^2 A^{-1})^{-1} \] (10)
where $\sigma_i^2, \sigma_j^2$ are the variances of $c_i, v_j$ respectively.

In this paper, an improved MMSE decoding algorithm can be obtained. Since the sparse matrix $A$ is an invertible matrix, the Eq.(10) can also be expressed by
\[ W_i^H = A \left[ A + (\sigma_i^2 / \sigma_j^2) I_4 \right]^{-1} \]
\[ = \left[ A + (\sigma_i^2 / \sigma_j^2) I_4 \right]^{-1} \]
\[ = (A + SNR^{-1} I_4)^{-1} \] (11)

where $I_4$ is the $4 \times 4$ unit matrix. As $A$ is a real symmetrical matrix, it can be eigen-decomposed by
\[ A = P \Lambda P^T = P \Lambda P \] (12)
where eigen-value matrix $\Lambda$ and orthogonal matrix $P$ can be written by
\[ A = \text{diag}(x-y, x-y, x+y, x+y) \] (13)
\[ P = \frac{\sqrt{2}}{2} \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \] (14)

Substituting Eqs.(12)-(14) into Eq.(11), we get
\[ W_i^H = \left[ P \Lambda P^T + SNR^{-1} PP^T \right]^{-1} = P \Lambda P \] (15)

where $V = \text{diag}(\xi_1, \xi_2, \rho, \rho)$, $\xi = (x-y+SNR^{-1})^{-1}$, $\rho = (x+y+SNR^{-1})^{-1}$.

Substituting Eq.(14) into Eq.(15), we get
\[ W_i^H = \frac{1}{(x+SNR^{-1})^{-1} - y^{-1}} \begin{bmatrix} 0 & 0 & -y \\ 0 & x+SNR^{-1} & y \\ 0 & y & x+SNR^{-1} \\ -y & 0 & 0 \end{bmatrix} \begin{bmatrix} x+SNR^{-1} \\ 0 \\ y \\ 0 \end{bmatrix} \]

(16)

By using Eq.(16) the weight matrix $W_i^H$ can be solved easily without matrix inversion operation. The computational complexity will be reduced obviously, and the computational numerical stability can also be improved.

3 Simulation Results

In order to verify the theory proposed above, some simulation experiments are carried out. The simulation conditions are, total data $1.6 \times 10^6$ bits, QPSK modulations. The simulation results curves are shown in Fig.1 and Fig.2. The left curves are the symbol error rate curves; the right curves are the bit error rate curves. From the two figures, we can find that the MMSE algorithm and the new MMSE algorithm proposed by this paper have same decoding performances (two curves are coincided), only the later has no matrix inversion operation, and is more effective in computation.

![Bit error probability of Jafarkhani QSTBC for two MMSE decoding algorithms](image)
4 Conclusions

By using the properties of the Jafarkhani QSTBC matrix, an effective MMSE decoding algorithm for Jafarkhani QSTBC in MIMO channel is proposed. The weight matrix can be solved directly without calculating the inverse matrix. The computational complexity is reduced obviously and the decoding performance is not affected. The simulation results have verified the theoretical analysis.

References


Brief Introduction to Author(s)

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