**Director Field in a Bipolar Configured Nematic Droplet**

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**Abstract**  Director field in a bipolar configured nematic droplet is analyzed numerically. Results of the calculation are expressed as the tilted angle of the director in the droplet, which balance between torque by elastic energy and the torque by external electrical field. The tilted angle is expressed as the function depending on latitude angle and relative radius $r / R$ in the spherical droplet. The result shows that the maximum difference of the tilted angle happens at 0.87R, where the tilted angle varies 12° before applying external field (0 V/$\mu$m.) and after applying electrical field (0.62 V/$\mu$m). If nematic droplet would be applied as a micro lens, its focus would vary because the refractive index changes due to the change of tilted angle. According to the calculation, maximum modification of refractive index is 0.036, if E7 would be adapted as the nematic phase in the droplet.

**Key words** PDLC; liquid crystals; nematic; droplet

Polymer dispersed liquid crystal (PDLC) have found its application in flexible displays and switchable windows. Such applications require dimension of the droplets to be several micro meters[1]. Recently, it is found that droplets in PDLC may gather together in some districts by special manufacture process, to form switchable period holographic gratings. Such an application requires dimension of the droplets less than 200 nm[2]. In spite of dimension of the droplets, whether in micrometers or sub-micro meter, both above PDLCs can be switched by external electrical field. As the switching property of PDLC is obtained by reorientation of the director of the liquid crystal in the droplets by electrical field, it is essential to study the behaviors of nematic droplets in electrical field.

As electronic-optical characters of PDLC originate from reorientation of director in the nematic droplets under external field, researches on reorientation behaviors of spherically confined liquid crystals never stop. Experiments show that, without external field, the average director in different droplets is randomly pointed, and by the weak field, average director starts turning and tends to point alone the field (when $\Delta \varepsilon > 0$, of course), in quite strong field, not only average director point alone the field, but also distribution of the director in the droplet concentrates alone the field[3]. Theoretical researches concert in the director distribution, namely director field, in a nematic droplet without field or in weak field. J. Vilfan, et al. analyzed director field of a nematic droplet without external field, they concluded that, the director distribution in the droplet possibly forms four types of configurations: 1) Bipolar configuration; 2) Star configuration; 3) Axial configuration; 4) Theoretically, there exists also toroidal configuration, though it is not observed till now[4]. B. Wu, et al. considered the shape of the droplets, based on the hypothesis that average director in the a droplet would turn to the direction where balanced elastic torques and the torques caused by external field. By such the balance condition, they obtained the expression of threshold field that director started turning[5].

The existing theories and calculations achieve the knowledge of average direction of the director in the droplet under external field, which explain most electron-optical character of PDLC that applied to the scattering devices like display and switchable window. For a big droplet, above consideration is quite enough, because molecular of liquid crystal near the boundary...
occupies small fraction volume referring the whole. However, for PDLC that forms holographic grades, diameter of the droplets is below 200 nm, the curvature of the droplets is bigger, hence director in the droplets can not be referred as uniformly oriented; another reason is, the molecular near the boundary occupies much more fraction of volume, effects of liquid crystal near the boundary can not be neglected. In latter case, average direction of the director cannot accurately describe the action of the droplets.

Additionally, our interesting is that the spherically shaped droplet acts like a lens. Its focal length changes if refractive index changes. External field will turn liquid crystal molecular, hence the director, so as to adjust refractive index and focus. Hence, it is necessary to reach the details about the director distribution under specified electrical field.

1 Configuration and Model

We start at the bipolar configuration with its symmetry axial alone external field, as shown in Fig.1a. Such a frame is built by the method specified in an American patent, or more frequently, by applying the weak external field\(^6\). It is deduced that, when the field increases, mean direction of the director keeps align alone the field and the director in the droplet tends to overcome elastic distortion to line alone the field that is schematically shown in Fig.1b. With small angel light scattering equipments, the deduction are proved by the author\(^7\). However, for detailed knowledge of relationship between external field and the director field, further analysis and calculation is required.

Let's initiate at the expression of Frank’s elastic free energy

\[
\Delta f = 2^{-1}\left[ k_{11} (\nabla \cdot \hat{n})^2 + k_{22} (\hat{n} \cdot \nabla \times \hat{n})^2 + k_{33} (\hat{n} \times \nabla \times \hat{n})^2 - 2^{-1} E^2 \Delta \varepsilon \cos^2 \alpha \right]
\]

where, \( \hat{n} \) is the director of liquid crystal, which is the function of position in the droplet; \( k_{11}, k_{22}, k_{33} \) are spay, twist and bend Frank elastic constant respectively, and for liquid crystal contains no chiral additives, \( k_{22} (\hat{n} \cdot \nabla \times \hat{n})^2 = 0 \); \( \Delta \varepsilon = \varepsilon_0 - \varepsilon_\perp \) presents the dielectric anisotropy of liquid crystal; \( \alpha \) is the angle between director \( \hat{n} \) and the external electrical field \( E \).

For the spherical droplet, we use spherical coordinate to convey the calculation. Let \( e_\theta \) be originated from the field, considering axial symmetry of the director field, as it is shown in Fig.1, then

\[
\hat{n}(r, \theta, \phi) = e_\theta \cos \theta (r, \theta, \phi) - e_\phi \sin \theta (r, \theta, \phi)
\]

where \( e_\theta \) and \( e_\phi \) are unit vectors in spherical coordinates, \( \theta \) is the latitude angle, and \( \phi \) is the longitude angle of the coordinate. \( \hat{n} \) is presented by \( \theta \), the angle between \( \hat{n} \) and \( r \), depending on position \( r \) and \( \theta \).

\[
\xi = \frac{1}{E \sqrt{\Delta \varepsilon}}
\]

Distribution of the director should balance between elastic torque and the torque by external electrical field, in other word, distribution of the director should minimize the total free energy in the droplet

\[
\delta \left\{ \frac{\Delta f\Omega}{\Delta \varepsilon} \right\} = 0
\]

where \( \Omega \) means total volume of the droplet.
Substituting Eq.(4) into Eqs.(1), (2) and (3), and applying Euler-Lagrange formula, we get a differential equation that describes distribution of the director in the droplet [8]

\[
\frac{\sin^2 \theta + \frac{k_{33}}{k_{11}} \cos^2 \theta}{2} \left( \frac{\partial^2 \theta}{\partial \rho^2} + \frac{\partial^2 \theta}{\partial \rho \partial \vartheta} \right) + \frac{\cos^2 \theta + \frac{k_{33}}{k_{11}} \sin^2 \theta}{2} \left( \frac{\partial^2 \theta}{\partial \vartheta^2} + \frac{\partial \theta}{\partial \vartheta} \cot \vartheta \right) - \sin 2 \theta \left( \cot^2 \vartheta - 1 \right) - \left( 1 + \frac{k_{33}}{k_{11}} \right) \cot \vartheta \sin^2 \theta + \frac{1 - (k_{33} / k_{11})}{2} \left[ \sin 2 \theta \left( \rho \left( \frac{\partial \theta}{\partial \rho} \right)^2 + \frac{\partial \theta}{\partial \rho} \cot \vartheta \right) - \left( \frac{\partial^2 \theta}{\partial \vartheta^2} + \frac{\partial \theta}{\partial \vartheta} \right) + 2 \rho \left( \frac{\partial \theta}{\partial \rho} \right) + 2 \cos 2 \theta \frac{\partial \theta}{\partial \vartheta} \frac{\partial \theta}{\partial \varrho} \right] - \frac{\left( \frac{R}{\xi} \right)^2 \rho^2 \sin(2(\theta - \vartheta))}{2} = 0
\]

(5)

where \( \rho = r / R \) is the dimensionless position vector, with \( R \) being real radius of the droplet.

Cross section of the droplet is divided by grid shown in Fig.2, and the differential Eq.(5) is converted to difference iterative procedure, as usually done.

\begin{equation}
\frac{1}{\xi} \sqrt{k_{11}} E = \left( \Delta \varepsilon \right)
\end{equation}

Assuming the radius of the droplet is 2 mm, considering the typical parameters of liquid crystals: \( k_{11} = 10^{-11} \text{N}, \Delta \varepsilon = 15\varepsilon_0, R / \xi = 0, 1, 2, 3, 4, 5 \) correspond to electrical field of 0, 0.14, 0.27, 0.41, 0.55, 0.69 V / \( \mu \text{m} \), respectively. However, if the radius of the droplet is 0.2 mm, the same strength of electrical field will result \( R / \xi = 0, 0.1, 0.2, 0.3, 0.4, 0.5 \), respectively.

**2 Calculation and Discussion**

Calculations proceed at elastic ratio \( k_{33}/k_{11} = 1, 1.5, 2 \), with \( R / \xi = 0, 1, 2, 3, 4, 5 \), respectively. Because minimal \( k_{33}/k_{11} \) is around 1 and maxim \( k_{33}/k_{11} \) is 2 for general liquid crystals, the calculations cover the general range of the elastic ratio \( k_{33}/k_{11} \) of liquid crystal material.

To have a quantity idea, let’s estimate strength of electrical field \( E \) expressed in term of \( R / \xi \), the ratio of radius of the droplet \( R \) to correlation length \( \xi \), according to Eq.(3), we get

\[
E = \frac{1}{\xi} \sqrt{k_{11}} \Delta \varepsilon
\]

(8)

Regarding the symmetry of the problem, we calculate only 1/4 cross section of the droplet, as shown in Fig.3. The resulted director of the liquid crystal molecular in the droplet is described by \( \theta \), the angle between \( \hat{n} \) and \( r \), varying with position. To make the results more readable, \( \theta \) is transferred to the tilted angle \( \alpha \), which is the angle between electrical field \( E \) and the director \( \hat{n} \).

The result of calculation is presented in Fig.4, where, Fig.4a is director distribution in term of tilted angle \( \alpha \) in the condition \( R / \xi = 0 \); Fig.4b is director distribution when \( R / \xi = 5 \).
The top values of the tilted angle for specified radius characterize the director field. The molecular near the center and border has zero tilted angle, the maximum tilted angle appears some where between center and border of the droplet, as shown in Fig.4.

Although top altitudes of the curves in Fig.4 are diverse for different radius and for different strength of field in the droplet, the shape of the curves is similar, which can be fitted by peak function. In other word, the curve would be redrawn according to the fitting parameters if top tilted angle and the position where top tilted angle appears would be known.

Fig.5 shows variation in relation to $r/R$ of the top tilted angle $\alpha_{\text{max}}$ of the director. Curve A exhibits change of $\alpha_{\text{max}}$ with position without external field, while curve B exhibits its change in the field strength of 0.69 V/μm, which is corresponded to $R/\xi=5$. Obviously, the field does not influent the director at center of the droplet, where $r/R=0$, for its direction has already oriented along the field. As to the director at the border, where $r/R=1$, because liquid crystal is strongly anchored, so the field can not turn the molecular of liquid crystal there, too. The peak variation of maximum tilted angle $\alpha_{\text{max}}$ appears at $0.78R$, as shown in Fig.6.

![Fig.4 Director distribution in term of tilted angle $\alpha$, result of calculations](image)

![Fig.5 Maximum tilted angle associated with specific radius in the droplet](image)

![Fig.6 Maximum difference of tilted angle in zero field and in the field specified as $R/\xi=5$](image)

A droplet acts like a lens. Its focal length changes if refractive index changes. External field turns director, so varies refractive index. Fig.6 more clearly shows the tilted angle difference between strong field and zero field. As it is shown that small elastic ratio is favorable to alter the tilted angle. It exhibits that maxim change of tilted angle is 12°, which correlates initial tilted angle being 35° without field, to the final tilted angle being 23° in electrical field. Corresponded refractive index are 1.585 and 1.549 respectively, when ordinary and extraordinary refractive index of the liquid crystal is 1.520 and 1.750, the refractive indexes of E7 made in E. Merck. Above simple calculation indicate that field strength being regulated from zero to 0.69 V/μm would alter refractive index of liquid crystals in the droplet up to 0.036. Such variation is suitable for faintly regulating focus of the micro lens.

3 Conclusion

Director field in a bipolar configured nematic
droplet is calculated numerically, based on that stable distribution of the director would be minimize the total free energy. Because director field could be expressed as the function of latitude angle and relative radius \( r/R \), and it could be fitted by peak functions, we extract the top tilted angle at specific radius of the droplet to indicate the director field. The calculation indicate that the maximal difference of tilted angle before applying external field and after applying external field happens at \( 0.87R \), and the variation of the tilted angle is 12° there, when the field varies from 0 to 0.62 V/\( \mu \)m.

In application of view, it is interesting that it is possible to modify refractive index of a lens. If nematic droplet is applied as a micro lens, according to our calculation, maximum modification of refractive index is 0.036.

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References


Brief Introduction to Authors

HUANG Zi-qiang (黄子强) was born in 1956. He is now an Associate Professor at UESTC. He graduated in 1982 from Radio Department, South China Engineering Institute, and received Physical Electronics Master degree in 1986 from UESTC. Then he turned to liquid crystal and display field. In 1992, he went to Calabria University in Italy, as a visiting scholar, studying liquid crystals. As the first author, he published 6 papers in some journals, such as: J. Appl. Phys., Molecular Crystals & Liquid Crystals, etc. He returned UESTC in 2000, and continued his research in liquid crystals, including electrical-optical characters of nanometer nematic micro droplets, large liquid crystal display system, bistable LCD and bistable OLED.

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