Comparison of Group-Buying Online Auction and Posted Pricing Mechanism in Uncertain Market

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Abstract Demand uncertainty is a key factor for the seller’s decision making, especially in the e-business environment, for the website to sell products through the online auction. In this paper, two kinds of demand uncertainties are considered: the consumer regime uncertainty and the inherent randomness of the market environment. Then, how to use a novel business model and group-buying auction (GBA) is analyzed in such a market environment. Based on the comparison of the GBA and the posted price mechanism, some conditions that favor the GBA are provided.

Key words group-buying auction; demand uncertainty; posted pricing mechanism

In recent years, with the development of e-business, online auctions have become popular venues for conducting business transactions. B2C auctions rapidly develop new sales channels via the Internet and extend the reach of the firm to previously inaccessible markets[1]. During online auctions, because the Internet is drawing on a broad spectrum of consumers, the sellers are less likely to be familiar with the consumer regime of their special products than the offline companies. Therefore, in the website’s decision making process, the demand uncertainty is an important factor that affects the websites’ strategy.

There are many new types of online auctions some of which are much different from the traditional auction. Among them, the group-buying auction (GBA), a homogeneous multi-unit auction, has recently received a lot of attention as part of the wave of innovative online market-based mechanisms. Although the GBA is widely used in the E-market, it is in great need to understand how to use the Group-buying properly for uncertain demand and find out the market conditions that favor the group-buying compared to the traditional posted pricing mechanism.

The rest of this paper is organized as follows. In section 2, we present an overview of demand uncertainties and related researches about GBA. In section 3, we build the model and describe the auction process. Section 4 considers the seller’s decision about price curve and compares the GBA mechanism with the traditional posted pricing mechanism to give the conditions that favor the Group-buying. In section 5, we summarize the paper and describe the future study directions.

1 Literature Survey

Generally, the demand uncertainties are composed of two parts: the consumer regime uncertainty and the inherent randomness of market environment. Dana proposes that when a seller faces uncertain consumer regime, the uniform price will not be a good pricing mechanism[2]. However, he does not study the scenario where there are fewer buyers in the high value regime and more buyers in the low value regime, which is considered in this paper.

In uncertain market, it is natural that the buyers have private information. Che and Gale show that when consumers have private information about both their budget constraints and their valuations then the monopolist’s optimal pricing strategy is to offer a menu of lotteries for its products[3]. By offering
different prices for different probabilities of obtaining the good the monopolist can profitably segment consumers even though segmentation on valuations alone is not profitable. Their model predicts both price dispersion and random delivery of the good, unlike other models of monopoly price dispersion.

Even in the case that the seller can clearly know about the consumer regime, because of the stochastic factors of market environment, the demand may still be uncertain. The seller may try to improve the demand forecast by utilizing the market signals observed during the sale. However, there are some differences between predictive demand and real one. Lo and Wu point out that a typical seller faces different types of risks, and among these, a key factor is forecast error. Dirim and Roundy quantify forecast errors, i.e., the differences between forecasts and actual demands. They lay out a scheme estimating the variance and correlation of forecast errors and modeling the evolution of forecasts over time. In this paper, we attempt to decrease the risk of uncertainty through a special pricing mechanism, group-buying online auction. So we need analyze and discover conditions that favor GBA compared to traditional posted pricing mechanism.

Because Group-buying auction is only adopted by some auction websites in recent years, the study on the GBA is rare. Chen et al. build a dynamic game model for the GBA, based on which they study the bidders’ optimal strategy. Kauffman and Wang conduct an experimental study of the GBA. They analyze the changes in the number of bids for Mobshop-listed products over various periods. These studies on consumer behavior in the GBA are very useful for the seller in optimizing the mechanism.

Meantime, it is more useful to find out conditions that GBA outmatches traditional posted pricing mechanism. Based on the empirical study of twelve GBA websites, Kauffman et al. suggest that the GBA websites that are oriented toward the B2B market should be better positioned for future growth. Anand and Aron compare the posted pricing mechanism with the GBA in different scenarios, e.g. demand uncertainty and economies of scale. Using simple analytic models, they provide the conditions that favor the GBA. Chen et al. analyze the seller’s profit for the group-buying auction when the economies of scale are considered, and proved that the group-buying auction will overmatch the posted pricing mechanism if the seller can always satisfy the bidders’ demand, which provides the Group-buying auction a suitable point. However, they all do not synthetically consider these two kinds of demand uncertainties, and because the GBA is a dynamic pricing mechanism, the demand uncertainty may be a key factor for decision making.

2 Modeling Framework

We consider a monopolistic retailer who will employ GBA as a pricing mechanism to reduce the risk of demand uncertainty. The auction process is described as follows.

At $T_0$, the seller posts the auction period $T_1$ and the price curve

$$PL = \left( \begin{array}{c} P \\ L \end{array} \right) = \left( \begin{array}{c} p_1, p_2, \ldots, p_m \\ l_1, l_2, \ldots, l_m \end{array} \right)$$

where $p_i > p_2 > \cdots > p_m$, $0 < l_1 < l_2 < \cdots < l_m$. In the price curve, the first row is the price levels and the second row is the upper limit amount of the bidders for the corresponding price level. We assume that only discrete bids are allowed, i.e., bidders can only bid $p_i$, $i \in [1, m]$.

The auction begins at $T_0$ and will end when either time reaches $T_1$ or there are $l_m$ bids. After the auction ends, the seller will deliver the products at $T_1$, and bidders will receive products at $T_2 = T_1 + T_L$, where $T_L$ is the transportation time of products. We suppose that the seller has enough products to supply.

In this paper, we consider the scenario that the seller has an estimate of the demand for his products but does not know it clearly. The monopolist will operate in one of two possible demand regime: the high value demand ($A$) and the low value demand ($B$). In $A$ and $B$ regimes, the CDF of bidders’ value distribution is $F_A(\cdot)$ and $F_B(\cdot)$ respectively. It is assumed that the bidders in regime $A$ will have a higher value than those in $B$. The arrive rates of $A$ and $B$ regimes are $\lambda_A$ and $\lambda_B$ respectively, where
\( \lambda_B > \lambda_A \), which means there are more consumers in the regime \( B \) than those in the regime \( A \). The probability that the regime \( A \) occurs is \( \alpha \) and \( B \) is \( 1 - \alpha \). So we need only consider two price levels, i.e., \( m=2 \) and the price curve

\[
PL = \left( \begin{array}{c} P \\ L \end{array} \right) = \left( \begin{array}{c} p_1, p_2 \\ l_1, l_2 \end{array} \right)
\]

Define

\[
\theta(v) = \begin{cases} 
q_1, & v \geq q_1 \\
q_2, & q_1 > v \geq q_2 \\
-\infty, & v < q_2
\end{cases}
\]

(1)

where \( v \) is a bidder’s value to the object. Let Strategy \( S \) denote the strategy that 1) if the bidder’s value, \( v \), is less than \( q_2 \), he bids nothing; 2) else, he bids \( \theta(v) \). With the similar assumption and proof to Chen et al. (2002)’s Theorem 1, it can be deducted that Strategy \( S \) is a weekly dominant strategy for the bidder. Hence we suppose the bidder will take Strategy \( S \) in the auction.

3 Seller’s Decision Process

In our setting, the seller must determine the price curve \( PL \) at \( T_b \). Because there are two kinds of demand uncertainties, the seller have to consider them sufficiently.

First the seller must set the price level \( p_1 \) and \( p_2 \). In posted pricing mechanism, the seller’s price level should be set to maximize his expected profit, i.e.

\[
p^* = \arg \max_p \pi(p)
\]

(2)

where \( \pi(p) = (p-c)D(p) \), with \( D(p) \) denoting the demand with price \( p \).

If the buyer’s arrival process is a Poisson process with arrival rate \( \lambda \)

\[
\text{Max } \pi(p) = \sum_{i=1}^{\infty} (p-c)D(p)\lambda T_i e^{-\lambda T_i} = \frac{\lambda T_i (p-c)(1-F(p))}{i!}
\]

Hence if the bidders are in regime \( A \), we set the price level as

\[
p_1 = \arg \max_p \lambda A T_i (p-c)(1-F_i(p))
\]

(3)

If the bidders are in regime \( B \), we set the price level as

\[ p_2 \in \arg \max_p \lambda B T_i (p-c)(1-F_i(p)) \]

(4)

Because the GBA price curve must fit both the regime \( A \) and \( B \), the seller sets the first price level at \( p_1 \), and the second at \( p_2 \).

After determining the price level, the seller has to choose appropriate \( l_1 \) and \( l_2 \). According to Anand and Aron, if the monopolist seller has enough supplying, he would set \( l_2 = \infty \) or a very larger number[10]. So, the seller only needs to consider \( l_1 \) which is very important for GBA mechanism to automatically adjust different price level according to various consumer regime. The following notation will be used:

\[
\lambda_i = \lambda_A, \quad \lambda_B = \lambda_A (1-F_A(p_i))
\]

\( \pi_A \): The seller’s expected profit with \( p_i \) in regime \( A \);

\( \pi_B \): With \( p_i \) and regime \( A \), the seller’s expected profit when the number of bidders is greater than \( l_1 \);

\( \pi_{A1} \): With \( p_i \) and regime \( A \), the seller’s expected profit when the number of bidders is less than \( l_1 \);

\( \Pr_{A1} \): With \( p_i \) and regime \( A \), the probability that the number of bidders are more than \( l_1 \);

\( \pi_B \): The seller’s expected profit with \( p_i \) in regime \( B \);

\( \pi_{B1} \): With \( p_i \) and regime \( B \), the seller’s expected profit when the number of bidders is greater than \( l_1 \);

\( \pi_{B0} \): With \( p_i \) and regime \( B \), the seller’s expected profit when the number of bidders is less than \( l_1 \);

\( \Pr_{B0} \): With \( p_i \) and regime \( B \), the probability that the number of bidders is less than \( l_1 \).

Theorem 1 The GBA will outperform the posted price mechanism, if the following conditions are satisfied

\[
\lambda_B l_1 < \lambda_A t \quad \frac{\lambda_A t}{(l_1 - \lambda_A t)^2} < 1 \quad \frac{\lambda_A l_1}{(l_1 - \lambda_A l_1)^2} < 1
\]

and

\[
\begin{cases}
\alpha \frac{\lambda_A t}{(l_1 - \lambda_A t)^2} (\pi_{A1} - \frac{p_2 - c}{p_1 - c} \frac{1-F_A(p_i)}{1-F_A(p_i)} \pi_{A1}) \\
< (1-\alpha)(1-\frac{\lambda_A t}{(l_1 - \lambda_A t)^2}) \pi_{B1} \\
\alpha(1-\frac{\lambda_A t}{(l_1 - \lambda_A t)^2}) (\pi_{A1} - \frac{p_2 - c}{p_1 - c} \frac{1-F_A(p_i)}{1-F_A(p_i)} \pi_{A1}) \\
> (1-\alpha)(1-\frac{\lambda_A l_1}{(l_1 - \lambda_A l_1)^2}) \pi_{B0}
\end{cases}
\]
Proof If choosing \( p_1 \) as the posted price, the seller’s expected revenue will be

\[
R_{p_1} = \alpha \pi_a + (1 - \alpha) \times 0 = \\
\alpha((1 - P_{r_a}) \pi_{a_1} + P_{r_a} \pi_{a_1})
\]

If choosing \( p_2 \) as the posted price, the seller’s expected revenue will be

\[
R_{p_2} = \alpha \pi_a + (1 - \alpha) \times 0 = \\
\alpha \left( \frac{p_2 - c}{p_1 - c} \left( \frac{1 - F_s(p_2)}{1 - F_s(p_1)} \right) \right) \pi_{a_2} + (1 - \alpha)(P_{r_a} \pi_{a_1} + P_{r_a} \pi_{a_1})
\]

The expected revenue will be

\[
R_{gba} = \alpha((1 - P_{r_a}) \pi_{a_1} + P_{r_a} \pi_{a_1})
\]

And with the GBA mechanism, the seller’s expected revenue will be

\[
R_{gba} = \alpha \left( \frac{p_2 - c}{p_1 - c} \left( \frac{1 - F_s(p_2)}{1 - F_s(p_1)} \right) \right) \pi_{a_2} + (1 - \alpha)(P_{r_a} \pi_{a_1} + P_{r_a} \pi_{a_1})
\]

As long as \( R_{gba} > R_{p_1} \) and \( R_{gba} > R_{p_2} \), GBA will outmatch the posted pricing mechanism. That is when the following conditions are met

\[
\begin{align*}
\alpha P_{r_a} \pi_{a_1} - \frac{(p_2 - c)}{p_1 - c} \left( \frac{1 - F_s(p_2)}{1 - F_s(p_1)} \right) \pi_{a_1} & < (1 - \alpha)(1 - P_{r_a}) \pi_{a_1} \\
\alpha(1 - P_{r_a}) \pi_{a_1} - \frac{(p_2 - c)}{p_1 - c} \left( \frac{1 - F_s(p_2)}{1 - F_s(p_1)} \right) \pi_{a_1} & > (1 - \alpha)P_{r_a} \pi_{a_1}
\end{align*}
\]

GBA will outperform the posted pricing mechanism.

Now, we will further analyze the relationship between \( l_1 \) and related market environmental parameters.

When \( \lambda t < l_1 < \lambda t \), and because \( l_1 = \infty \) or larger enough, according to Chebyshev inequality

\[
P_{r_a} < \frac{\lambda t}{(l_1 - \lambda t)(1 - F_s(p_2))t} = \frac{\lambda t}{(l_1 - \lambda t)^2}
\]

because \( F_s(p_2) = 0 \), and

\[
Pr_{r_a} < \frac{\lambda t}{(l_1 - \lambda t)^2}
\]

hence, we can get the conclusion.

If the seller can get the appropriate \( l_1 \) satisfies Theorem 1, then he should choose the GBA as pricing mechanism, otherwise he will select the posted prices.

4 Conclusions and Future Study

In this paper we study the Group-buying auction in the uncertain market. We consider the uncertainty of the consumer regimes and the stochastic arrival process of the buyer’s. In such a market environment, we compare the Group-buying with the posted price mechanism to find out the conditions that the Group-buying outmatches the posted price mechanism.

In our model, we only find some conditions about \( l_1 \), with which the GBA will be advantageous over posted pricing mechanism. However, if we can determine the optimal price level of the GBA, and then compare it with the optimal posted price to get the optimal \( l_1 \), it will be more useful. Additionally, we assumed that the seller can get enough products to supply. When he must decide the order quantity during the auction, it will be much different. Hence it is more interesting and general to consider how to determine the best price curve and order quantity together.

References


Brief Introduction to Author(s)

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