Purchase Contract Management for Fashion Products

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Abstract This paper develops a two-stage purchase contract for fashion products to manage the buyer’s cost. In addition to require a commitment, the purchase contract allows the buyer to adjust the commitment at a later stage. We model the buyer’s problem and obtain explicit formulae to describe the buyer’s optimal behavior. We analyze the effect of parameters on the buyer’s cost and prove that such contract can decrease the buyer’s cost numerically.

Key words purchase contract; supply chain; quantity flexibility; forecast update

This paper develops a two-stage purchase contract for fashion products in a two-party supply chain consisting of a supplier (manufacturer) and a buyer (distributor or retailer). Fashion products, for instance apparel products, are characterized by long lead-times, short selling seasons and high demand uncertainties. Suppliers typically want buyers to commit themselves to purchasing stable large quantities early. However, due to the high demand uncertainties, to reduce the risks of shortage or overage, buyers typically want to delay ordering decisions so as to incorporate updated demand forecasts [1]. Further, buyers need flexibility so that if customer demand becomes much higher than initially anticipated, they can receive more products from suppliers. But, on the other hand, if customer demand becomes much smaller than initially anticipated, buyers want the ability to cancel initial order. Hence, the suppliers’ objectives are in direct conflict with the buyers’ desires.

Quantity flexible (QF) contracts may provide one trade-off solution for such conflict. In a QF contract, the supplier requires the buyer to make a commitment earlier in order to ensure markets and mitigate demand uncertainty. As a reward, the supplier provides the buyer with flexibility to adjust order quantity later.

Various QF contracts have been proposed. Donohue develops a supply contract where the supplier has two production modes: normal mode and fast mode [2]. Fast mode allows the buyer to order additional products so as to take advantage of updated demand forecasts. Donohue identifies efficient conditions that coordinate the channel to maximize total profit. Bassok and Anupindi study total minimum quantity commitment contract, and derive the buyer’s optimal purchase policy [3]. This is a single product contract where the buyer makes total minimum quantity commitment before planning horizon and decides order quantity period by period. The supplier offers discounts as a reward for the commitment. Emmons and Gilbert analyze effects of the return policy on profits [4]. Such policies are often used to encourage larger initial orders under the condition of uncertain demands, long production lead-times, and short selling seasons [4]. Bengtsson uses options to realize return policy and analyzes coordination of contracts over one period [5].

Different with above literatures, in this paper we develop a two-stage contract model with both upward and downward flexibility. The flexibility is realized by options. Options give the buyer the right to increase (for call options) or decrease (for put options) initial order. At the beginning of planning horizon, the buyer decides how many products to order initially and how many options to purchase. Demand forecast is updated during the supplier’s long lead-time. Before selling season starts, the buyer decides the type and quantity of the options to exercise. We study the buyer’s ordering policy and expected cost for such a contract.
1 Model

We consider a two-stage purchase contract between a supplier and a buyer. The buyer purchases products from the supplier at wholesale price, and sells to customers at fixed retail price that is exogenously specified (see Fig.1). We assume that the lead-time is long and the supplier has one manufacturing opportunity.

The contract provides an opportunity both to increase and decrease commitment by options. If the options are exercised as calls, the buyer pays unit exercise price for each option exercised. If the options are exercised as puts, the buyer can get back exercise price for each option exercised. If the options are exercised as calls, the buyer pays unit increase and decrease commitment by options. If the opportunity.

Events in our model have the following sequence

From \( t_0 \) to \( t_1 \): 1) The supplier offers a contract menu of \((w, \omega_0, \omega_n)\). 2) The buyer places initial order \( Q_0 \) (\( Q_0 \geq 0 \)) and purchases options \( q_0 \) (\( q_0 \geq 0 \)).

From \( t_1 \) to \( t_2 \): 1) The buyer calculates holding cost for unsold products or shortage cost for unsatisfied demand, then selling season ends.

\[
\begin{align*}
&\text{Contract menu} \\
&\text{(w, \omega_0, \omega_n)} \\
&\text{Delivery } Q_0+q_e. \\
&\text{Supplier} \\
&\text{Lead-timer} \\
&\text{Demand forecast} \\
&\text{update} \\
&\text{Selling reason} \\
&\text{Time} \\
&\text{Options } q_e. \\
&\text{Options exercised } q_e. \\
&\text{Buyer} \\
&\text{Initial order } Q_0 \\
&\text{Options } q_0. \\
&\text{Fig.2 Events sequence}
\end{align*}
\]

Let \( D \) represent the random demand in selling season. We assume that \( X \) is uniformly distributed over the interval \( [\gamma - n, \gamma + n] \) (\( \gamma > n \gg 1 \)) and \( D \) follows uniform distribution over the interval \([x - m, x + m]\) (\( m \gg 1 \)) given \( X = x \). \( X \) is unknown at \( t_0 \), and specialized as \( x \) at \( t_1 \). The pdf. and cdf. of \( X \) as well as those of \( D \) are as follows.

\[
\begin{align*}
&f_X(x) = \frac{1}{2n+1} \quad x \in [\gamma - n, \gamma + n] \\
&F_X(x) = \frac{(x - \gamma + n)}{2n+1} \quad x \in [\gamma - n, \gamma + n] \\
&f_{Q_k}(\xi) = \frac{1}{2m+1} \quad \xi \in [x - m, x + m] \\
&F_{Q_k}(\xi) = \frac{(\xi - x + m)}{2m+1} \quad \xi \in [x - m, x + m]
\end{align*}
\]

1.1 Optimal Solution at \( t_1 \)

Depending on the value of the variable \( X \), we derive 3 cases at \( t_1 \). We analyze the 3 cases and show the cost functions and optimal solutions as below.

Case 1: \( Q_0 + q_0 < x \)

In this case, even the buyer exercises all \( q_0 \) options as calls, the final order quantity \( Q_0 + q_0 \) cannot meet the possible demand. Consequently, shortage occurs and the cost function is

\[
G_1^0(Q_0, q_0, q_e) = w_{ue}\cdot q_e + p\int_{x-n}^{x+m} \frac{\xi - Q_0 - q_0}{2m+1} d\xi
\]

Obviously, the optimal solution is \( q^*_e = q_0 \). Then the minimum cost function is

\[
G_1^1(Q_0, q_0, q_e) = w_{ue}\cdot q_0 + p\int_{x-n}^{x+m} \frac{\xi - Q_0 - q_0}{2m+1} d\xi
\]

Case 2: \( Q_0 - q_0 > m \)

In this case, since the minimum quantity of the final order quantity \( Q_0 - q_0 \) exceeds the possible
maximum demand, the buyer unavoidably carries holding cost for overage stock. Corresponding cost function can be written as below.

\[ G_i^*(Q_0, q_0, q_e | x) = wq_e + h \int_{t_{-\infty}}^{t_{+\infty}} \frac{Q_0 + q_e - \xi}{2m + 1} d\xi \]  \hspace{1cm} (3)

It is clear that minimum cost realized at \( q_e^* = -q_0 \), and the minimum cost function is

\[ G_i^*(Q_0, q_0, q_e | x) = w(-q_0) + h \int_{t_{-\infty}}^{t_{+\infty}} \frac{Q_0 - q_0 - \xi}{2m + 1} d\xi \]  \hspace{1cm} (4)

Case 3: \( Q_0 + q_0 + m \geq X \geq Q_0 - q_0 - m \)

In this case, either overage or shortage may occur. The cost function of this case is as below.

\[ G_i^*(Q_0, q_0, q_e | x) = \begin{cases} G_i^*(Q_0, q_0, q_e | x) & \text{if } q_0 \geq q_e \geq 0 \\ G_i^*(Q_0, q_0, q_e | x) & \text{if } 0 \geq q_e \geq -q_0 \end{cases} \]  \hspace{1cm} (5)

where

\[ G_i^*(Q_0, q_0, q_e | x) = wq_e + h \int_{t_{-\infty}}^{t_{+\infty}} \frac{Q_0 + q_e - \xi}{2m + 1} d\xi - p \int_{t_{-\infty}}^{t_{+\infty}} \frac{\xi - Q_0 - q_e}{2m + 1} d\xi \]
\[ + h \int_{t_{-\infty}}^{t_{+\infty}} \frac{Q_0 + q_e - \xi}{2m + 1} d\xi + p \int_{t_{-\infty}}^{t_{+\infty}} \xi - Q_0 - q_e d\xi \]
\[ G_i^*(Q_0, q_0, q_e | x) = wq_e + h \int_{t_{-\infty}}^{t_{+\infty}} \frac{Q_0 + q_e - \xi}{2m + 1} d\xi - p \int_{t_{-\infty}}^{t_{+\infty}} \frac{\xi - Q_0 - q_e}{2m + 1} d\xi \]
\[ + h \int_{t_{-\infty}}^{t_{+\infty}} Q_0 + q_e - \xi d\xi + p \int_{t_{-\infty}}^{t_{+\infty}} \xi - Q_0 - q_e d\xi \]

Taking the derivative of \( G_i^*(Q_0, q_0, q_e | x) \) and \( G_i^*(Q_0, q_0, q_e | x) \) with respect to \( q_e \) and equate to zero respectively, we can get the optimal solution for case 3 is to exercise \( q_e^* \) options, and

\[ q_e^* = \begin{cases} q_e, & \text{if } Q_0 + q_0 + y(w_{\text{in}}) \leq x \leq Q_0 + q_0 + m \\ x - y(w_{\text{in}}) - Q_0, & \text{if } Q_0 + y(w_{\text{in}}) \leq x \leq Q_0 + q_0 + y(w_{\text{in}}) \\ 0, & \text{if } y(w_{\text{in}}) + Q_0 \geq x \geq y(w) \hspace{1cm} (6) \\ x - y(w) - Q_0, & \text{if } Q_0 - q_0 + m \leq x \leq Q_0 - q_0 + y(w) \\ -q_0, & \text{if } Q_0 - q_0 + m \leq x \leq Q_0 - q_0 + y(w) \end{cases} \]

where

\[ y(x) = \frac{(h - p)m + (2m + 1)x}{h + p} \]

The corresponding minimum cost function for case 3 can be written as

\[ G_i^*(Q_0, q_0, q_e | x) = \begin{cases} G_i^*(Q_0, q_0, q_e | x), & \text{if } Q_0 + q_0 + y(w_{\text{in}}) \leq x \leq Q_0 + q_0 + m \\ G_i^*(Q_0, q_0, x - y(w_{\text{in}}) - Q_0 | x), & \text{if } Q_0 + x - y(w_{\text{in}}) - Q_0 \leq x \leq Q_0 + q_0 + y(w_{\text{in}}) \\ G_i^*(Q_0, q_0, 0 | x), & \text{if } y(w_{\text{in}}) + Q_0 \geq x \geq y(w) \hspace{1cm} (7) \\ G_i^*(Q_0, q_0, x - y(w) - Q_0 | x), & \text{if } Q_0 - q_0 + y(w) \leq x \leq Q_0 + y(w) \\ G_i^*(Q_0, q_0, -q_0 | x), & \text{if } Q_0 - q_0 + y(w) \leq x \leq Q_0 - q_0 + y(w) \end{cases} \]

Combining the optimal cost functions in case 1 to case 3, we obtain integrated optimal cost function as below.

\[ G_i^*(Q_0, q_0, q_e | x) = \begin{cases} G_i^*(Q_0, q_0, q_e | x), & \text{if } Q_0 + q_0 + m \leq x \leq \gamma + n \\ G_i^*(Q_0, q_0, x - y(w_{\text{in}}) - Q_0 | x), & \text{if } Q_0 + x - y(w_{\text{in}}) - Q_0 \leq x \leq Q_0 + q_0 + m \\ G_i^*(Q_0, q_0, 0 | x), & \text{if } y(w_{\text{in}}) + Q_0 \geq x \geq y(w) \hspace{1cm} (8) \\ G_i^*(Q_0, q_0, x - y(w) - Q_0 | x), & \text{if } Q_0 - q_0 + y(w) \leq x \leq Q_0 + y(w) \\ G_i^*(Q_0, q_0, -q_0 | x), & \text{if } Q_0 - q_0 + y(w) \leq x \leq Q_0 - q_0 + y(w) \end{cases} \]

1.2 Optimal Solution at \( t_0 \)

At \( t_0 \), the buyer’s problem is to find optimal values of \( Q_0 \) and \( q_0 \) to minimize the cost function \( G(Q_0, q_0) \).

\[ \min G(Q_0, q_0) = \min \{ wQ_e + wq_0 + E[G_i^*(Q_0, q_0, q_e | x)] \} \]  \hspace{1cm} (9)
Subject to \( Q_0 \geq 0, q_0 \geq 0 \)

where

\[
E[G_i(Q_0, q_0, q_0)] = \int_{y+w_0}^{y+w_0+\gamma} G_i(Q_0, q_0, y-q_0, x) f_x(x)dx + \int_{y-w_0}^{y-w_0-\gamma} G_i(Q_0, q_0, x) f_x(x)dx + \int_{0}^{y-w_0-w_0} G_i(Q_0, q_0, x-y(w_0) - Q_0) f_x(x)dx + 
\]

Taking the derivative of \( G(Q_0, q_0) \) with respect to \( Q_0 \) and equating to zero, we get:

\[
m(p + h)Q_0 = \frac{1}{2} B(w, w_n, m, h, p)q_0 + D(w, h, p, \gamma, m, n) \tag{10}
\]

where

\[
B(w, w_n, m, h, p) = (w + w_n)(2m + 1) + 2m(h - p)
\]

\[
D(w, h, p, \gamma, m, n) = -(2m + 1)(n + \frac{1}{2})w + pm(\gamma + n) + hm(\gamma - n)
\]

Taking the derivative of \( G(Q_0, q_0) \) with respect to \( q_0 \) and equating to zero, we have

\[
A(w, w_n, m, h, p)q_0 = B(w, w_n, m, h, p)Q_0 + C(w, w_n, w_0, h, p, \gamma, m, n) \tag{11}
\]

where

\[
A(w, w_n, m, h, p) = (w - w_n)(2m + 1) + 2m(h + p)
\]

\[
C(w, w_n, w_0, h, p, \gamma, m, n) = \frac{[(h - p)m + (2m + 1)w_n] \gamma}{2(h + p)} - (2m + 1)(\gamma - n)w_n - \frac{[(h - p)m + (2m + 1)w_n] \gamma}{2(h + p)} - 2hm(\gamma - n) + 2pm(\gamma + n) - (p + h)m^2
\]

Let \( Q_0' \) and \( q_0' \) represent the solution for

Eq.(10) and (11). If \( Q_0' \geq 0, q_0' \geq 0 \), then the optimal solution is \( Q_0' = Q_0' \) and \( q_0' = q_0' \). Note that the buyer prefers to no options when \( q_0' \leq 0 \). If \( Q_0' \geq 0, q_0' < 0 \), the optimal solution is \( Q_0 = D(w, h, p, \gamma, m, n) / (m(p + h)) \) and \( q_0' = 0 \). For \( Q_0' < 0 \), since it has less meaningful indications for practice, we do not discuss it in this paper.

### 2 Numerical Examples

We analyze the effect of parameters on the buyer’s cost. Parameter settings are as follows: \( \gamma = 1000, n=200, m=150, w=100, w_0=10, w_{on}=120, p=200, h=70 \). When we study a parameter, values of the others are fixed. To save figure space, we analyze two relationships in each figure.

In Fig.3, we plot the corresponding cost as function of \( w_0 \) for various \( p \). Horizontal coordinate represents \( w_0 \) and vertical coordinate represents cost. Points that have equal \( p \) values are connected and form independent curves. We take \( w_0 \) from 10 to 90 and \( p \) from 130 to 220. As \( w_0 \) increases, the cost increases and finally hits the maximum value. This is as expected since the increase in \( w_0 \) raises the flexibility cost and when the flexibility is too expensive the buyer prefers no flexibility. The maximum cost in Fig.3 is the no flexibility contract case. Note that using options can reduce the buyer’s cost compared with no flexibility contracts. On the other hand, the cost increases as \( p \) increases.

![Fig.3](image-url)

In Fig.4, we show the relationship between cost and \( w_0 \) for various \( p \). We increase \( w_{on} \) from 105 to 195 with a step size 10. We observe that the cost increases with \( w_{on} \); while if \( w_{on} > 175 \), the cost decreases. It is
reasonable that the cost increases with exercise price of call options $w_{eu}$. The reason for the decrease in cost is as following: When $w_{eu}>175$, the buyer should make a large commitment to reduce probable expensive exercise cost of call options. To mitigate the risk from large commitment, the buyer will enhance flexibility, which reduces the buyer’s expected cost. As $h$ increases, we observe that the buyer’s cost increases. Reducing holding cost is a way to decrease cost.

3 Conclusions

In this paper we modeled a two-stage flexible purchase contract where the buyer can adjust initial order both upward and downward. We formulated the buyer's cost function and determined the buyer's optimal policy. We analyzed the effect of parameters on the buyer’s cost and proved that such contract can decrease the buyer’s cost numerically.

References


Brief Introduction to Author(s)

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