A Simplified Power Adaptation Scheme for Wireless Orthogonal Frequency Division Multiple Access*

FU Yin-fei, WU Shi-qi
(School of Communications and Information Engineering, UESTC Chengdu 610054 China)

Abstract Orthogonal frequency division multiple access (OFDMA) is a promising access technology based on multiple carrier transmission. In this paper, we develop a power and bits allocation scheme for multiuser OFDMA to minimize the system margin according to channel state information (CSI). Compared with other OFDM methods, our scheme has higher system capacity and better BER performance.

Key words OFDM; OFDMA; adaptive; water filling; power allocation

Recently, orthogonal frequency division multiplexing (OFDM) has been received more and more attention because of its ability to eliminate large ISI, multipath delay and high spectral efficiency. The principle of OFDM is to divide the frequency selective multipath wireless channels into a set of orthogonal flat fading subchannels, so that a high-speed data stream is split into a set of lower rate streams.

OFDMA is a kind of air interface access based on OFDM. In OFDMA, the carriers are divided into subgroups, especially named subchannels in this paper, each constituting a set of subcarriers. Data on each subcarrier all contribute to the OFDM symbol, so the performance of the OFDM system will be decided by the fading on subcarriers. The occurrence of bit errors is normally concentrated in a few severely faded subcarriers, while other subcarriers could exhibit low bit errors or no errors. So the high bit-error probability in those severely faded subcarriers will dominate and deteriorate the total performance of systems. Moreover, because different users often experience mutually independent frequency selective fading of the channel, thus subcarriers in deep fading for one user may not be in deep fading for other users. Therefore, if the optimization of subchannels for each user and bits and energy for each subcarrier can be achieved according to instantaneous channel state information (CSI) of each user, the total performance of the OFDM system will be greatly improved.

In this paper, we present a multiuser subcarrier and energy allocation for OFDMA aiming to maximize the system margin with a fixed data rate. To maximize a predetermined-rate margin, this equivalently minimize the total transmit energy\(^1\). In Ref.[2], C.Y. Wong et al. investigated a dynamic subcarrier and power allocation scheme which aims to minimize to the total transmit power under a given set of user data rates. However, the efficiency and the convergence rate of Wong’s algorithm depend critically on the step size and the initial point of the iterative searching, and possibly results in different computations time each searching. For systems with a large number of subcarriers, the algorithm becomes prohibitively expensive. In this paper, a simpler scheme will be proposed to avoid a great of number of searching.

1 System Model

The structure of the adaptive multiuser OFDMA-based system under consideration is shown in Fig.1.

We assume that the CSI is perfect for our adaptive multiuser allocation algorithm and that feedback is error free. The system has \(K\) users and \(N\) subcarriers. The \(k\)th user has a given total data rate equal to \(B_k\) bits
per OFDM symbol. The data from $K$ users are sent into adaptive modulator block. The $K$ subchannels containing a set of subcarriers are allocated to the $K$ users and the number of bits and energy are loaded on each subcarrier according to adaptive subcarrier and power allocation algorithm. The subchannels for each user are dynamically computed and allocated every time under all users joint CSI and rate constraints. According to bits and energy for each subcarrier, the corresponding modulation scheme is selected. OFDMA access scheme requires that each subcarrier cannot be shared by more than one user and must be used exclusively by one user. We introduce the parameter $b_{k,n}$ as the number of bits of the $k$th user on the $n$th subcarrier. So there is $b_{k,n} \leq 0$, $b_{k,n} = 0$ for all $k' \neq k$.

From Eq.(1) we can easily get the energy function

$$E_{k,n} = \frac{\Gamma}{g_{k,n}} (2^{b_{k,n}} - 1)$$ (2)

So the multiuser Energy Minimization problem can be formulated as

$$\min_{b_{k,n}} \sum_{k=1}^{K} \sum_{n=1}^{N} \frac{\Gamma}{g_{k,n}} (2^{b_{k,n}} - 1) \quad \text{subject to}$$

$$\sum_{n \in S_k} b_{k,n} = B_k \quad \text{for all } k$$ (3b)

$$b_{k,n} \geq 0 \quad \text{for all } k \text{ and } n$$ (3c)

$$S_i \cap S_j = \emptyset \quad i \neq j, i \text{ and } j \in \{1,2,\cdots,K\}$$ (3d)

$$S_i \cup S_j \cup S_1 \cup \cdots \cup S_K = \{1,2,\cdots,N\}$$ (3e)

where $b_{k,n} \in \{0, 1, 2, \cdots, M\}$ is the set of all possible integer bit values for each subcarrier, $S_k$ denotes the set of subcarrier assignment for the $k$th user.

2 Multiuser Subcarrier and Bit Allocation

To make Eq.(3) tractable, we can consider an alternative optimization problem by relaxing the $b_{k,n}$ to be a real number within $[0,1,\cdots,M]$. To formulate the constraint Eq.(3d) and Eq.(3a) mathematically, the new parameters, $\rho_{k,n}$ for all $k$, $n$ with the values within $[0,1]$ will be introduced. The new optimization problem is then given

$$\min_{b_{k,n},\rho_{k,n}} \sum_{k=1}^{K} \sum_{n=1}^{N} \rho_{k,n} (2^{b_{k,n}} - 1) \quad \text{subject to}$$

$$\sum_{n \in S_k} b_{k,n} = B_k \quad \text{for all } k$$ (4b)

$$\sum_{n=1}^{N} \rho_{k,n} = 1 \quad \text{for all } n$$ (4c)

$$b_{k,n} \geq 0 \quad \text{for all } k \text{ and } n$$ (4d)
The additional variable $\rho_{k,n}$ can be considered as the time-sharing factor for user $k$ of the $n$th subcarriers. Another equivalent interpretation of $\rho_{k,n}$ is that $\rho_{k,n}$ represents the fraction amount of bandwidth assigned to user $k$ in the $n$th subcarriers.

To satisfy Eqs.(3d) and (3e) in the original optimization problem, the values of $\rho_{k,n}$ have

$$
\rho_{k,n} = \begin{cases} 
1 & \text{if } b_{k,n} \neq 0 \\
0 & \text{if } b_{k,n} = 0 
\end{cases}
$$

(5)

It is easy to show that the values of $\rho_{k,n}$ in Eq.(5) satisfy the constraint Eq.(4c). The optimization problem of Eq.(4) is a convex optimization problem. So the original optimization problem of Eq.(3) can be considered as the bound of the modified optimization problem of Eq.(4), but the modified optimization problem of Eq.(4) is more tractable.

Using Lagrangian multiplier in Eq.(4), we can get

$$
L = \sum_{k=1}^{K} \sum_{n=1}^{N_k} \frac{\rho_{k,n}}{b_{k,n}} \left(2^{\frac{E_{k,n}}{\Gamma}} - 1\right) + \sum_{k=1}^{K} \lambda_k \left(B_k - \sum_{n=1}^{N_k} b_{k,n}\right) + \sum_{n=1}^{N} \beta_n \left(1 - \sum_{k=1}^{K} \rho_{k,n}\right)
$$

(6)

Differentiating Eq.(6) with respect to $b_{k,n}$ and setting it to 0, we get

$$
b_{k,n} = \rho_{k,n} \left(\log_2 M_k + \log_2 \frac{\Gamma b_{k,n}}{E_{k,n}}\right) \quad \text{for all } k
$$

(7)

where $M_k = \frac{\lambda_k}{\ln 2}$. Considering $E_{k,n}$ in Eq.(4a), we can obtain

$$
\frac{E_{k,n}}{\rho_{k,n}} + \frac{\Gamma b_{k,n}}{\rho_{k,n}} = M_k \quad k = 1, 2, \ldots, K
$$

(8)

Eq.(8) are similar to the classical single-user water-filling solutions, it can be called as multiuser water-filling solutions. $M_k$ is the optimal water-filling level for user $k$. Eq.(8) shows that when $E_{k,n} > 0$ for user $k$, there exists some constraint $M_k$ to make Eq.(8) hold true. If $E_{k,n}$ is negative, the subcarrier $n$ cannot be used by user $k$ and should be excluded from the optimization process.

3 Discrete Multiuser Adaptive Algorithm

Unfortunately, there is no explicit method to calculate the water-filling level $M_k$ for all users. A large number of searching for $M_k$ is needed for the optimal solutions [2], it will be prohibitively expensive in practical systems, especially when the number of subcarriers increases. Furthermore, Eq.(8) is obtain by relaxing subcarrier to be shared among users and bits to be real numbers, which is not matched with the requirements of OFDMA. So we develop a suboptimal discrete algorithm that allocates each subcarrier to only one user and limits the bits to the integer. Generally, our algorithm is solved through two steps: subcarrier assignment for users and adaptive bit and power allocation for each user.

In the first step, a specific set of subcarriers is exclusively allocated to only one user. In a single user water-filling solutions, the flat energy distribution exhibits negligible loss with respect to the optimal water-filling energy distribution as long as the energy is only poured into subcarriers exactly the same as water-filling does [5]. So we prefer to take into account the flat energy distribution among users who are sharing one subcarrier, which is easy to show that the user who has the best CNR exhibits a bit higher spectral efficiency than any other users. If the individual user has no rate constraints to satisfy, we would allocate each subcarrier to the user who has the highest gain on that subcarrier.

In some cases, however, some users whose channel states are in a good condition over the whole band might be allocated more subcarriers than the maximal number of subcarriers that they really need, while other users who have no best CNR over the subcarriers cannot be assigned any subcarrier. This will lead to the high outage probability. To reduce the possibility of outage, our goal is to compute the minimal number of subcarriers for as many users as possible under the constraint of each user’s data rate. This problem is equivalent to solve the rate maximization for each possible user. This is to say, to maximize the data rate under the maximal permitted transmit power constraints for a given set of subcarriers, i.e.

$$
\text{Maximize} \quad \sum_{k=1}^{K} \log_2 \left(1 + \frac{E_{k,n}}{\Gamma} \right) \quad \text{s.t.} \quad \sum_{n=1}^{N_k} E_{k,n} = E_k
$$

(9a)

(9b)
where $E_i$ is the maximal system energy allocated to the $k$th user. This is the well-known single user Rate Maximization water-filling algorithm \[^4\] Its solution is solved using the iteration algorithm, and the $i$th water-filling level can be found as follows

$$M_i = \frac{1}{N_k - i} (E_i + \Gamma \sum_{s=1}^{N_k-1} \frac{1}{g_{s,i}}) \quad (10)$$

where $N_k$ is the initial total number of subcarriers allocated to the user $k$ based on above-mentioned discussions. Eq.(10) culminates with $N_k - i$ for the first value of $i$ that does not cause a negative energy on $E_{k,o}$ to occur, then leaving the energy as $E_{k,o} = M_i - \Gamma / g_{k,i}$.

Our algorithm is stated as follows.

1) Set the number of users to be allocated is $K' = K$.

2) Allocate each subcarrier to the user who has the best CNR among all the users sharing this subcarrier. Set the total number of users to be successfully allocated is $K'$. Set $N_k$ for the $k$th user.

3) For $k = 1, 2, \ldots, K$.

(1) Sort $g_{k,i}$ in decreasing order according to $n$ and set $index = 1$.

(2) Compute the iteration Eq.(10) with respect to $j$ from 1 to $index$ until $E_{k,j} \leq 0$.

(3) Compute $b_{k,j}$, $1 \leq j \leq index$.

(4) If $\sum\limits_{j} b_{k,j} < B_k$, then $index = index + 1$ and go to 3.2, or else go to the next 3.5.

(5) Update $N_k = index$.

4) Release the unused subcarriers, set its total number is $N'$. If $N' \neq 0$, set $K' = K - K^*$, then go back to 2; if yes, then go to end.

The $N_k$ is the updated subcarrier assignment for the user $k$.

Based on the result from the first step, we can adaptively allocate bits and energy to users on the specific set of subcarriers according to instantaneous CSI. Actually, this is equivalently single user energy minimization problem, which belongs to a class of discrete optimization known as separable convex discrete resource allocation (SCDRA) problem \[^6\]. This problem can be solved based on the greedy optimization, which means that each increment of additional information to be transmitted is placed on the subcarrier that would require the least incremental energy for its transmission. Furthermore, if the bits distribution $B \in \mathbb{R}$ is a solution to the continuous margin maximization problem, the vector $R(B)$, which is rounded according to Eq.(11), is the result of greedy optimization \[^4\]. Here, $B = (b_1, b_2, \cdots, b_N)$.

$$R(b_i) = \begin{cases} \frac{\lfloor b_i \rfloor}{\lfloor b_i \rfloor + 1} & \text{if fractional part of } b_i < 0.5 \\ \lfloor b_i \rfloor & \text{if fractional part of } b_i > 0.5 \end{cases} \quad (11)$$

### 4 Simulation Results

In this section, we present the simulation results of our proposed subcarrier and power allocation scheme. The simulations are preformed of an adaptive uncoded multiuser OFDM system with 512 subcarriers. We assume that the CSI is perfect at the receiving end and users are independently distributed, therefore they experience mutually independent frequency selective fading of the channel. The number of users is in 4 and 20. The channel model is based on ITU-R M1225 Rayleigh multipath model for urban areas \[^7\].

We consider a system that employs BPSK, QPSK, 8QAM, 16QAM and 64QAM, which are used to carry 0, 2, 3, 4 and 8 bits/OFDM symbol for each subcarrier, depending on the number of bits allocated. The total of 200 to 512 bits is transmitted in each OFDM data frame, depending on the number of users. The required BER is $10^{-4}$, equivalent to 6.6 dB of the gap $\Gamma$.

Fig.2 compares the outage probability of two kinds of algorithm with and without searching the minimal number of subcarriers for each user. We can find that the outage possibility of the multiuser margin maximization scheme with searching has about 80% lower than that without searching.

Fig.3 shows BER of our multiuser adaptive subcarrier and power allocation compared with other static multiuser subcarrier allocation methods. They are presented as follows:

OFDM-Continued-FDMA: each user is continually assigned a predetermined set of subcarriers and can only use those subcarriers exclusively.

OFDM-Interleaved-FDMA: because there is a high correlation between the channel gains of adjacent subcarriers in a frequency selective fading channel, subcarriers assigned to a user are interleaved with other
users’ subcarriers in order to avoid the case in which all the subcarriers of one users experience a deep fade. From Fig.3, we can find that our proposed scheme has at least 3~6 dB higher than other fixed subcarrier allocation methods.

The latter aims to reduce the outage probability of users. Simulation results show that the improvement is significant. The new scheme has higher system capacity and better BER performance.

5 Conclusions
In this paper, we develop a new simplified subcarrier and power adaptation scheme to maximize system margin under the fixed rate constraints for multiuser OFDMA systems. Our two-step scheme firstly computes the minimal number of subcarriers available for each user. Then optimal discrete bit and energy assignments are computed for users based on the results of the first step. The key of our proposed scheme is to allocate a subcarrier to the user who has the best CNR and to search the minimization of subcarriers needed to achieve the rate requirements.

References

Brief Introduction to Author(s)
FU Yin-fei (付寅飞) was born in 1974. He received the B.S and M.S. degrees both in Electronic Engineering at University of Electronic Science and Technology of China, respectively in 1997 and 2000. He is currently pursuing the Ph.D. degree in electronic engineering. His interests include adaptive modulation, OFDM, MIMO, adaptive signal processing.

WU Shi-qi (吴诗其) was born in 1938. He is now a professor and Ph.D. advisor at School of Comm. Info. & Engineering at University of Electronic Science and Technology of China. His interests include satellite communications, wireless mobile communications and signal processing.