Analysis of Intermediate Product Price and Social Welfare in the Vertical Merger of Enterprises

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Abstract According to the different products that two upper enterprises produce, this paper analyzes the transfer pricing of intermediate product and the effect of social welfare of vertical merger under imperfect competition market. The conclusions are: the transfer pricing of intermediate product is equal to marginal cost; the perfect outputs that upper enterprises sell to the market before and after the vertical merger of enterprises are the same; the product price in the integrated enterprises may be higher, lower or equal to the price in non-integrated enterprises.

Key words vertical merger; intermediate product; transfer pricing; social welfare

With the development of modern economy, the merger of enterprises is not only the important growth method of the enterprise, but also the strategic tool with which a nation may adjust the industrial structure and guarantee its social welfare. Used as a kind of enterprise’s market behavior, the enterprise merger plays a key role in extending the scale of enterprise, increasing the property effectively and improving the economic performance. The merger of enterprise is divided into horizontal merger, vertical merger, mixed merger and so on. Many countries pay attention to the occurrence of the horizontal merger, for horizontal merger can destroy the competition and therefore form high monopoly situation. The definition of vertical merger is that advantage enterprise merges other enterprises that produce the product related to the advantage enterprise to extend the production scale, and therefore obtains the scale economy. Based on Ref.[1], under one enterprise merges two upper enterprises, this paper analyzes the transfer pricing of intermediate product and the effect of social welfare of vertical merger under imperfect competition market. The difference of intermediate product price between the vertical merger and non-vertical merger is that the two upper enterprises can price their own product which sold to the downstream enterprise, then the downstream enterprise prices the final product which can bring the perfect profit before the merger, but after the merger, based on the final product demand function and target of perfect profit, the downstream enterprise decides the intermediate product price[2].

1 Analysis of Intermediate Product Price and Social Welfare in the Vertical Merger of Enterprises

1.1 The Model Assumption
1) There are three enterprises 1, 2, 3 in the market. Enterprises 1, 2 are upper enterprises, they have their own fixed marginal cost $C_1$, $C_2$ and produce different products, their outputs are $Q_1, Q_2$. The market demand functions of the two intermediate products are

\[ P_1 = f_1(Q_1) = a_1 - b_1Q_1 \quad P_2 = f_2(Q_2) = a_2 - b_2Q_2 \]

2) The scale guerdons of the downstream enterprise 3 are constant, its production function is $Q = f(Q_1, Q_2)$, i.e., enterprise 3 uses the products that enterprises 1, 2 produce to produce the final product, the final product demand function is $P = P(Q)$, suppose $P$ is a linear function $P = a - bQ$, the downstream enterprise 3 merges the upper enterprises 1, 2.
3) The amounts of the intermediate products sold to the outermarket are \( Q_{m1}, Q_{m2} \). Enterprises 1, 2, each sells the products to the enterprise 3 at the prices of \( P_1, P_2 \). The prices of intermediate products is decided by enterprise 3.

4) Enterprise 3's cost which doesn't include the cost of intermediate products is \( TC_3(Q_1, Q_2) \).

5) Enterprises 1, 2, 3 have the ration to pursue the perfect profits. The vertical merger cost includes the law expense, consultation cost and so on. The profits that brought by the vertical merger are higher than the costs of vertical merger, otherwise, enterprises 1, 2, 3 don't have the tendency to merge.

1.2 The Intermediate Product Prices after Vertical Merger of Enterprise 1, 2, 3 under the Fixed-Proportional Output Function

Suppose that the production function is the fixed-proportion function, by choosing appropriate unit, devoting each an unit main factor to get an unit. Assume that \( Q_1 = Q_2 = Q_1 \), i.e., enterprise 3 produces one unit final product needing enterprises 1, 2, each one unit intermediate product. The following will discuss, respectively, the incomes profits, intermediate products price, outputs of enterprises 1, 2, 3. The total income of enterprises 1, 2, 3 are

\[
TR_i = P_iQ_i + P_{m1}Q_{m1} \quad TR_3 = P_2Q_2 + P_{m2}Q_{m2} \quad TR_3 = PQ
\]

respectively, after vertical merger. The profits of enterprise 1 is \( \pi_1 \), then

\[
\pi_1 = P_iQ_i + P_{m1}Q_{m1} - TC_1(Q_1 + Q_{m1})
\]

Set \( \pi' = 0 \), we conclude that the perfect outputs of enterprise 1 after vertical merger obey the following

\[
\frac{\partial \pi_1}{\partial Q_i} = \frac{dTR_1}{dQ_i} - \frac{\partial}{}TC_i(Q_i', Q_{m1}') = P_i - C_i = 0
\]

i.e., the premium product price of enterprise 1 is equal to its marginal cost

\[
P_i = C_i \quad (2)
\]

Likewise, marginal profit of the products that enterprise 1 sells to the outer market is equal to its marginal cost

\[
MR_{m1}(Q_{m1}') = C_i \quad (3)
\]

According to the similar discussion of enterprise 2, we conclude that

\[
P_2 = C_2 \quad (4)
\]

\[
MR_{m2}(Q_{m2}') = C_2 \quad (5)
\]

According to the Eqs.(2), (4), we find that the transfer pricing of intermediate product produced by enterprises 1, 2 are equal to their own marginal cost. Because there exists the imperfect competition market of intermediate products and enterprises have the ration to pursue the perfect profits, the outer market prices of intermediate products \( Q_1, Q_2 \) must be higher than their marginal cost, we have \( P_1 < P_{m1}, P_2 < P_{m2} \).

Suppose the profit of enterprise 3 are \( \pi_3 \), then

\[
\pi_3 = PQ - P_1Q_1 - P_2Q_2 - TC_3(Q_1, Q_2) = (a-b)Q - P_1Q_1 - P_2Q_2 - TC_3(Q_1, Q_2)
\]

Because of \( Q = Q_1 = Q_2 \), Eq.(6) can be changed into

\[
\pi_3 = (a - P_1 - P_2)Q - bQ - TC_3(Q) \quad (7)
\]

To obtain the perfect yields of enterprise 3 after vertical merger, set \( \pi'_3 = 0 \), i.e.,

\[
(a - P_1 - P_2) - 2bQ - MC_3(Q) = 0 \quad (8)
\]

From the above, we can easily get that the marginal cost of enterprise 3 is equal to the sum of prices of intermediate products bought by enterprise 3

\[
MC_3(Q) = P_1 + P_2 \quad (9)
\]

According to Eqs.(8), (9), we can have

\[
(a - P_1 - P_2) - 2bQ = (P_1 + P_2) = 0 \quad (8)
\]

The solution is

\[
Q' = \frac{(a-2P_1-2P_2)}{2b} \quad (10)
\]

According to Eqs.(2), (4), (10), the perfect yields of enterprise 3 are

\[
Q' = \frac{(a-2C_1-2C_2)}{2b} \quad (11)
\]

Then the perfect price is

\[
P' = a - bQ' = a - b\left(\frac{a-2C_1-2C_2}{2b}\right) = \frac{a - b}{2}a + C_1 + C_2 = \frac{1}{2}a + C_1 + C_2
\]

The total profit of enterprises 1, 2, 3 after vertical merger is
\[ \pi = PQ + P_m(Q_{o1}) + P_m(Q_{o2} - TC_1(Q_{o1} + Q_{o2}) - \]
\[ TC_2(Q_{o2}) - TC_1(Q_{o2}) = PQ + \]
\[ P_m(Q_{o1}) + P_m(Q_{o2} - TC_1(Q + Q_{o2}) - \]
\[ TC_2(Q + Q_{o2}) - TC_2(Q) \]

Set \( \pi' = 0 \), we can have the perfect price of the final production that the enterprise produces after vertical merger
\[ P' = C_1 + C_2 + MTC_3(Q') \]

We can also derive
\[ MR(Q') = C_1 + C_2 + MTC_3(Q') \]
\[ MR(Q_{o1}^{'}) = MTC_1(Q' + Q_{o1}^{'}) \]
\[ MR(Q_{o2}^{'}) = MTC_2(Q' + Q_{o2}^{'}) \]

Eq.(11) indicates that after vertical merger, enterprises’ marginal income is equal to marginal cost in accordance with Refs.[3,4]. From the above, we can not only have the transfer pricing of intermediate product under imperfect competition market of intermediate product, but also have that the intermediate products’ price must be equal to marginal products’ cost so as to maximize the profits of vertical merger under the condition the marginal cost of upper enterprises is known. We are told that the enterprises before vertical merger and after vertical merger arrive at the perfect profits, when upper enterprises’ marginal cost is known and the price of intermediate products equals to the marginal cost is known.

1.3 The Analysis of Social Welfare before and after Vertical Merger under the Condition that the Outputs of the Three Enterprises are Equal

1) Assume that the profits of enterprises 1, 2, 3 after vertical merger are \( \pi_{i1}^{'}, \pi_{i2}^{'}, \pi_{i3}^{'}, \) respectively, then
\[ \pi_{i1}' = PQ' + f_1(Q_{o1}) + Q_{o1} - TC_1(Q + Q_{o1}) = \]
\[ PQ' + (a_i - b_i Q_{o1}) - TC_1(Q + Q_{o1}) \]
\[ \pi_{i2}' = PQ' + f_2(Q_{o2}) + Q_{o2} - TC_2(Q + Q_{o2}) = \]
\[ PQ' + (a_2 - b_2 Q_{o2}) - TC_2(Q + Q_{o2}) \]
\[ \pi_{i3}' = (a - b)Q' - PQ' - PQ - TC_3(Q) \]

The perfect profits of enterprises 1, 2, 3 after vertical merger is
\[ \pi^* = \frac{a^2 - 4(C_1 + C_2)^2}{4b} + \frac{a_1^2 - C_1^2}{4b_1} + \frac{a_2^2 - C_2^2}{4b_2} - \]
\[ TC_1(Q' + Q_{o1}) - TC_2(Q + Q_{o2}) - TC_3(Q) \]

where
\[ Q' = \frac{a - 2C_1 - 2C_2}{2b} \]
\[ Q_{o1}' = \frac{a_1 - C_1}{b_1} \]
\[ Q_{o2}' = \frac{a_2 - C_2}{b_2} \]

2) Each profit of enterprise 1, 2, 3 are \( \pi_{i1}, \pi_{i2}, \pi_{i3} \), i.e.,
\[ \pi_{i1} = a_i - b_i(Q + Q_{o1}) - TC_1(Q + Q_{o1}) \]
\[ \pi_{i2} = a_2 - b_2(Q + Q_{o2}) - TC_2(Q + Q_{o2}) \]
\[ \pi_{i3} = (a - b)Q - TC_3(Q) - [a_i - b_i(Q + Q_{o1})]Q - \]
\[ [a_2 - b_2(Q + Q_{o2})]Q \]

Enterprise 1, 2, 3 maximize their profit respectively and then the three enterprises’ profits are added, we can have
\[ \pi'' = (a_i - C_i)[2b_1(a + a_i) - (a_i - C_i)(b_i)] + \frac{a_1^2 - C_1^2}{4b_1} - \]
\[ TC_1(Q'' + Q_{o1}''') - TC_2(Q'' + Q_{o2}''') - TC_3(Q'') \]

where
\[ Q'' = \frac{a_1 - C_1}{2b_1} \]
\[ Q_{o1}''' = \frac{a_2 - C_2}{2b_2} \]

Thus, between the profits before and after vertical
\[ \pi^* - \pi'' = \frac{a^2 - 4(C_1 + C_2)^2}{4b} + \frac{a_1^2 - C_1^2}{4b_1} + \]
\[ TC_1(Q'' + Q_{o1}''') + TC_2(Q'' + Q_{o2}''') + TC_3(Q'') - \]
\[ TC_1(Q' + Q_{o1}') - TC_2(Q + Q_{o2}) - TC_3(Q) \]

(14)

Comparing \( Q_{o1}' \) with \( Q_{o1}''' \), \( Q_{o2}' \) with \( Q_{o2}''' \), and \( Q' \) with \( Q'' \), we conclude that \( Q_{o1}' = Q_{o1}''' \), \( Q_{o2}' = Q_{o2}''' \), \( Q' < Q'' \) or \( Q' > Q'' \) or \( Q' = Q'' \).

Because social welfare is increased[1], i.e., the profits after vertical merger are higher than the profits before vertical merger. I.e., \( \pi^* - \pi'' > 0 \). Assume \( Q' < Q'' \), then
\[ TC_1(Q'' + Q_{o1}''') - TC_1(Q' + Q_{o1}') > 0 \quad i = 1,2 \]

Since \( \pi^* - \pi'' > 0 \), we can make sure the relationship
among \( a, b, C_1, C_2, a_1, b_1 \) or \( a, b, C_1, C_2, a_2, b_2 \).

Analogously, when \( Q' > Q'' \), \( Q' = Q'' \), we can also make sure the relationship among \( a, b, C_1, C_2, a_1, b_1 \) or \( a, b, C_1, C_2, a_2, b_2 \).

From the above, we conclude the follows.

1) If \( a, b, C_1, C_2, a_1, b_1 \) or \( a, b, C_1, C_2, a_2, b_2 \) are given, we can make sure the relationship among them, then we can decide the outputs of final product before and after vertical merger and even the price of final products.

2) The final product price after merger may be higher or lower or equal to the price before merger, but the profits after merger must be higher than the profits before merger, i.e., social welfare is increased after vertical merger.

2 Summary

From the above discussion about the transfer pricing of intermediate product and the analysis of social welfare before and after vertical merger, we can have:

1) Under the conditions that the marginal costs of the upper enterprises 1, 2 are known, each perfect intermediate product price of enterprises 1, 2 is equal to their own marginal cost after vertical merger. The enterprise can easily decide the perfect price of intermediate product when the marginal cost is known, then the time value can be obtained.

2) The perfect yields of enterprises 1, 2 which are sold to the intermediate product market after merger are the same as the perfect yields before merger, i.e., vertical merger doesn’t have the influence on the perfect output of intermediate product sold to the market.

3) Social welfare is increased after vertical merger. The perfect price of final products after merger maybe higher, lower or equal to the perfect price before merger. On the purpose that analyzing the question easily. This paper makes some assumption which maybe not completely right. So the work that needs us to do is on the one hand we should analyze question deeply, on the other hand we should match the actual process, embodying these assumptions.

References


Brief Introduction to Author(s)

YU Juan (于娟) was born in 1979. She is now pursuing her Master’s Degree at UESTC. Her main research interest is in the Intermediate Product Price.

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