Analysis of Coupling between DR and Microstrip*

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Abstract The coupling between dielectric resonator (DR) and microstrip is theoretically analyzed in this paper. The equivalent circuit and corresponding components’ expression are achieved. Also, the formula of the coupling ratio is deduced, which is the critical parameter to calculate the values of components. The analysis of the paper can be used in modeling and strict design in DR applications such as dielectric resonator oscillators(DROs) and DR filters. At the end of the paper, an example of DRO design based on the analysis above is given. The coincidence between modulation and measure shows the significance of the paper.

Key words dielectric resonator; microstrip; coupling; dielectric resonator oscillator

Dielectric resonator(DR) is made of high-dielectric coefficient and low-loss dielectric material. The coupling between DR and microstrip is often used in oscillators and filters. So it is important to research its equivalent circuit.

1 Value of Equivalent Circuit

When a cylindrical DR, in TE_{01δ} mode, is approaching to the microstrip, the interaction between two magnetic fields, which come from DR and from the exciting current of the microstrip, is magnetic effect, i.e., mutual inductance (see Fig.1)\[1\]. This effect will excite the electromotive force in the microstrip. The following is the analysis of the transmission characteristic of the microstrip by the action of the electromotive force. Suppose \( l_1 < l_2 \).

Because the microstrip’s electric field is mainly centralized under the microstrip and it is orthogonal with the electric field \( E_0 \) of the TE_{01δ} mode, there isn’t any inter-coupling between them. In fact, with high \( \varepsilon_r \), the dielectric of DR has little effect on the microstrip.

When \( l_1 \) is longer than \( \lambda_g/8 \), we can reasonably suppose that the coupling between the microstrip, \( l_2-l_1 \), and DR is so poor that it can be ignored. Then the whole network can be divided into two parts. Move the reference plane of \( x=-l_1 \) and \( x=l_2 \) into \( x=0 \), which is the center plane of DR. So the expression for the two-port scattering parameter is given by

\[
[S'] = \begin{bmatrix}
\frac{2\beta\gamma}{\gamma + j\beta\delta} & 1 \\
1 + \frac{2\beta\gamma}{\gamma + j\beta\delta} & 1 \\
1 + \frac{2\beta\gamma}{\gamma + j\beta\delta} & 1 \\
\frac{2\beta\gamma}{\gamma + j\beta\delta}
\end{bmatrix}
\]  

(1)

where the phase constant is

\[
\beta^2 = \omega^2LC \\
\gamma = \frac{LL}{m_0}Gio
\]
\[ \eta = \frac{1}{\beta} \int_{\beta}^{\beta} L_{\alpha}(x) \cos(\beta') x dx \]

\[ \delta = \int_{\beta}^{\beta} \int_{\beta}^{\beta} L_{\alpha}(x) \sin(\beta') x dx \int_{\beta}^{\beta} L_{\alpha}(x) \cos(\beta') x dx - \int_{\beta}^{\beta} \int_{\beta}^{\beta} L_{\alpha}(x) \cos(\beta') x dx \int_{\beta}^{\beta} L_{\alpha}(x) \sin(\beta') x dx \]

So the network can be equivalent as a series resistor. Its value is given by

\[ Z = \frac{4\beta^2 \eta}{\gamma + j\beta \delta} \] (2)

It can be analyzed that the series resistor can be equivalent as a shunt-resonant circuit. See Fig.1b. The values of the components can be expressed as

\[ R \approx 2\beta Z_0, \quad L \approx \frac{2\beta Z_0}{\omega_0 Q_0}, \quad C \approx \frac{Q_0}{2\beta Z_0 \omega_0} \] (3)

where \( \beta \) is the coupling coefficient between the DR and microstrip, \( Z_0 \) is the characteristic impedance of the microstrip, \( \omega_0 \) is the resonant frequency of DR, \( Q_0 \) is its unloaded quality factor. The coupling coefficient is defined as

\[ \beta = \frac{Q_0}{Q_e} = \frac{Q_0 k^2}{2Z_0} \] (4)

The coupling ratio is defined as

\[ k^2 = \frac{2\beta Z_0}{Q_0} = \frac{2\omega}{Z_0} W \left[ \frac{1}{\beta} \left( \int_{\beta}^{\beta} E_e(t) d t \right) \cos(\beta' x) d x \right]^2 \] (5)

where \( E_e(x) \) is the inducing electric field produced by the resonator’s alternative magnetic field along the microstrip of \( \beta \) direction, \( W \) is the average magnetic energy stored in DR. If \( E_e(x) \) and \( W \) are given, the value of the coupling ratio, coupling coefficient and components in Eq.(3) can be obtained.

### 2 Solution of Coupling Ratio

\( E_e(x) \) and \( W \) can be solved by numerical method. Z component of Hertz vector scale in all parts of Fig.1c can be expanded in the form of Fourier series as

\[
\begin{align*}
P_1 &= \sum_{p} A_{p1} J_0(k_p r) \sin \beta_p' \left[ z - (h + d) \right] \\
P_2 &= \sum_{p} A_{p2} J_0(k_p r) \sin \beta_p' \left[ z + \delta_p \right] \\
P_3 &= \sum_{p} A_{p3} J_0(k_p r) \sin \beta_p' \left[ z + (h + \delta_p) \right] \\
P_4 &= \sum_{p} C_{p4} H_0(k_p r) \sin \beta_p' \left[ z - (h + d) \right] \\
P_5 &= \sum_{p} C_{p5} H_0(k_p r) \sin \beta_p' \left[ z + (h + \delta_p) \right]
\end{align*}
\]

where \( A_{p1}, A_{p2}, A_{p3}, B_{p4}, C_{p4} \) and \( \delta_p \) are the undetermined constants, and there is a relation:

\[ k_p^2 = \left( \frac{\omega}{c} \right)^2 - \beta_p'^2 = \left( \frac{\omega}{c} \right)^2 \varepsilon - \beta_p'^2 \]

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where \( \varepsilon \), is the DR’s dielectric constant, \( \varepsilon \), is the dielectric constant of the substrate. So the average magnetic energy stored in the DR is expressed as

\[ W = \frac{e_0 \omega^2 \mu^2}{2} \int_{\beta}^{\beta} \int_{\beta}^{\beta} \left( \frac{\partial P}{\partial r} \right)^2 \left( \frac{\partial P}{\partial r} \right)^T r dr dz \]

for the relation

\[ J_0(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{2^{2n}(n!^2)} \]

the average magnetic energy stored in the DR is

\[ W = e_0 \omega^2 \mu^2 \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} (-1)^{m+n} \frac{A_{m,n}^2}{2^{2(m+n)-1}} \frac{\pi^2}{(m+1)!n!(n-1)!} \left[ (2m+n-1) \right] \]

\[ \left\{ L - \frac{1}{2\beta_p} \sin 2\beta_p'(h + L + \delta_p) \right\} (h + L + \delta_p) \]

\[ - \sin 2\beta_p'(h + \delta_p) \}

the electric field can be expressed as

\[ E_e(x) = E_0 \cos \theta = E_0 \frac{r_0}{\sqrt{r_0^2 + x^2}} \]

\[ j \omega \mu \frac{r_0}{\sqrt{r_0^2 + x^2}} \frac{\partial P}{\partial r} \]

\[ - j \omega \mu \frac{r_0}{\sqrt{r_0^2 + x^2}} \sum_{q} B_q H_0 \left( k_q \sqrt{r_0^2 + x^2} \right) \sin \beta_q' d \]

Hankel function is defined as

\[
H_0(x) \approx j \frac{1}{\pi} 2 \ln \frac{x}{2}
\]

thus,

\[ H_0' \left( k_x \sqrt{r_0^2 + x^2} \right) = \frac{dH_0(k_x r)}{dr} \mid_{r=\sqrt{r_0^2 + x^2}} = j \frac{2}{\pi} \sqrt{\frac{r_0^2 + x^2}{x^2}}
\]

substituting the above equation for Eq.(10) and \( E_e(x) \) can be obtained as
substituting Eqs.(9) and (11) for Eq.(5) and the coupling ratio can be expressed as

\[
k^2 = \frac{8\omega}{Z_0 W \pi^2} \left( \frac{\sum B_q k_q \sin \beta_{k_q} d \times}{x^2} \right) \left( \cos \beta' x dx - \frac{\pi}{4\beta \sin \beta' l} \right)^2 \]

(12)

The unknown numbers in Eq. (12) can be solved by the following method.

\[
\begin{align*}
\frac{\tan \beta_{k_p}' (d - L)}{\beta_{k_p}'} + \frac{\tan \beta_{k_p}' h}{\beta_{k_p}'} - \\
\frac{\beta_{k_p}'}{\beta_{k_p}'} \tan \beta_{k_p}' (d - L) \tan \beta_{k_p}' h = 0 \quad (13)
\end{align*}
\]

Eq.(13) are substituted by Eq.(7) and the unknown numbers of \(k_p, k_q, \beta_{k_p}', \beta_{k_q}', \beta_{k_p}, \beta_{k_q}, \delta_p\) and \(\delta_q\) can be solved. By the following equations

\[
\begin{align*}
\sum_{p} A_{2p} k_p^2 J_{y_p}(k_p a) J_{y_p} = B k_p^2 H_y(k_p a) I_y \quad (14) \\
\sum_{q} A_{2q} k_q^2 J_{y_q}(k_q a) J_{y_q} = B k_q^2 H_y(k_q a) I_q \quad (15)
\end{align*}
\]

where

\[
J_{pq} = \frac{1}{\beta_{k_p}'^2 - \beta_{k_q}'^2} \left\{ \begin{array}{l}
\sin \beta_{k_p}' (h + \delta_p) \sin \beta_{k_q}' d \\
\sin \beta_{k_q}' h \sin \beta_{k_p}' h 
\end{array} \right\} \times 
\]

(16)

the values of \(A_{2p}\) and \(B_q\) can be obtained. By the following equations

\[
\begin{align*}
A_{1p} &= -A_{2p} \sin \beta_{k_p}' (h + L + \delta_p) \sin \beta_{k_q}' (d - L) \\
A_{3p} &= A_{2p} \sin \beta_{k_p}' (h + \delta_p) \sin \beta_{k_q}' h \\
C_q &= -B_q \sin \beta_{k_q}' d \sin \beta_{k_q}' h 
\end{align*}
\]

the values of \(A_{1p}, A_{3p},\) and \(C_q\) can be obtained. Then \(W\) and \(E_{1}(x)\) can be obtained. From Eq.(12), the coupling ratio \(k^2\) can be expressed.

3 Applications

By the analysis of the article and Math CAD software, a C-band reflection-type dielectric resonator oscillator (DRO) has been developed by author. Fig. 2 shows its micro strip topology. The HJ-FET in the DRO is NE325S01 from NEC and \(Z_0\) of \(l_1\) and \(l_2\) is \(80 \Omega\), \(l_1=6.38 mm(\theta=90^\circ)\), \(l_2=4.32 mm(\theta=60^\circ)\). The other dimensions of micro strips(\(Z_0=50 \Omega\)) are: \(l_3=4.24 mm, l_4=4.1 mm, l_5=2.93 mm\).

Fig.2 DRO microstrip topology

The measured phase noise of the DRO is \(-94 \text{ dBc/Hz@10 kHz}\) and \(-118 \text{ dBc/Hz@100 kHz}\), which is very close to the results of simulation by ADS software. Fig. 3 shows its simulated plot of phase noise.

Fig.3 Simulated results of DRO phase noise

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