Phenomenological Theory for Giant Magnetoresistance Oscillation in Magnetic Multiplayer

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Abstract  A phenomenological theoretical model for magnetic multilayer based on Boltzmann equation and Fuchs-Sondheimer theory is studied. An oscillatory transmission coefficient ($T'$) is introduced into the boundary conditions to simulate the alternate ferromagnetic and anti-ferromagnetic coupling between the magnetic layers. The transmission coefficient has an oscillatory factor relating to the thickness of space layer. Other effects such as interface roughness on $T'$ are also taken into account. The numerical results for [Fe/Cr] multilayer agree with the experimental data very well, both in the period and range of oscillation, which is leaded by the dependence of giant magnetoresistance(GMR) on layer thickness of space layer.

Key words  magnetic multilayer;  giant magnetoresistance (GMR);  phenomenological theory

Electrical transport properties in nanostructure have experimentally and theoretically importance. The interest in such researches is stimulated by possible applications in microelectronic devises such as giant magnetoresistance (GMR) sensor and magnetic random access memory (MRAM). One interesting property of the electrical transport in magnetic multilayer composed of ferromagnetic separated by nonmagnetic layers is the spacer dependence of GMR oscillation, which is generally believed to be caused by the interlayer exchange interaction. That is, with very thin spacer, the alternative occupation of ferromagnetic and antiferromagnetic coupling between adjacent magnetic layers with a changing spacer thickness account for that oscillatory phenomenon.

Two theoretical approaches to the GMR have been developed. One is the quantum-statistical based on Kubo formalism$^{[1,2]}$. Another is phenomenological theory based on Boltzmann equation and Fuchs-Sondheimer theory for size effects in electronic transport$^{[2-5]}$. Phenomenological theory provides a basic physical conception of the spin dependence scattering in or between the magnetic layers, but can’t describe the oscillatory phenomenon properly.

Some researchers pointed out that the quantum interference of electron waves could lead to the oscillations in the dependence of the GMR on the layer thickness$^{[1]}$. These oscillations can be related to the oscillation behavior of the electron transmission coefficient $T$, which denotes the possibility of an electron passing from one magnetic layer to another. The ordinary phenomenological approach describes that the GMR effect well in the case that the sublayers are much thick, for the quantum effect can be neglected when the bulk electron mean free paths are much shorter than the sublayer thickness. Under this quantum neglect $T$ is usually treated as a constant. This is why the phenomenological method cannot exposit an oscillatory change in GMR. In this paper we relate $T$ to the interlayer exchange coupling by taking the thickness of space layer and the roughness of interfaces into consideration. The GMR values of Fe/Cr multilayers are calculated by this new model and the results agree with the experimental data. Based on this model the oscillation phenomena can be interpreted reasonably.

1 Theoretical Model

The phenomenological model for GMR is an
extension of the Sondheimer-Fuchs resistivity model of a thin film. The geometry of sandwich including mixing region is presented in Fig.1 which shows two ferromagnetic (FM) layers separated from a non-magnetic layer (NM) by a mixing region (M) on two sides. A static electric field is applied along the x-axis parallel to the interfaces. The dash line in the middle of the NM layer is an artificial boundary at which the quantization axis for the spin electron changed. In the case it is assumed that the electron transport through the multilayer structure is governed by the semi-classical Boltzmann equation

\[ \frac{e}{m}(E + vB)\Delta_v f + v\Delta_x f = \frac{(f - f_0)}{\tau} \]

where \( f \) is the electron distribution function.

Fig.1  Geometry of the FM/NM/FM sandwich used in the theoretical description.

The distribution function is decomposed into two parts: the equilibrium distribution function \( f_0(v) \) and a small perturbation distribution function induced by external fields

\[ f^s(z, v) = f_0^s(v) + g^s(z, v) \]

where \( s \) represents the spin directions of the electron (\( \uparrow \) for spin up and \( \downarrow \) for spin down)

Substituting Eq.(2) into the Boltzmann equation and retain only the linear terms in the perturbation, one obtains

\[ \frac{\partial g^s}{\partial z} + \frac{g^s}{\tau' v_s} = \frac{eE}{mv_s} \frac{\partial f_0}{\partial v_s} \]

where \( e \) and \( m \) denote the electron charge and effective mass, \( \tau' \) is the relaxation times for spin-up and spin-down electrons. For convenience, separate \( g \) into two parts: \( g^+ \) for electrons with positive \( v_s \) and \( g^- \) for electrons with negative \( v_s \). The general solution for Eq.(3) is

\[ g^+ = eE r' \frac{\partial f_0}{\partial v_s} [1 + F^+ exp(-\frac{z}{\tau' v_s})] \]

\[ g^- = eE r' \frac{\partial f_0}{\partial v_s} [1 - F^- exp(-\frac{z}{\tau' v_s})] \]

where \( I \) notes the regions described in Fig.1 \( (A,B,C,D,E,F) \). In this equation the only unknowns are the values of \( F \), which can be calculated through the boundary conditions at the interfaces between regions

\[ g^A = T^A g^A \quad \text{at} \quad z = -c \]

\[ g^B = T^B g^B \quad \text{at} \quad z = +c \]

\[ g^C = T^C g^C + R^C g^D \quad \text{at} \quad z = -b \]

\[ g^D = T^D g^D + R^D g^C \quad \text{at} \quad z = +b \]

\[ g^E = T^E g^E + R^E g^F \quad \text{at} \quad z = +a \]

\[ g^F = T^F g^F + R^F g^E \quad \text{at} \quad z = +b \]

where \( p \) is the specularity factor of faces, \( T^s \) and \( R^s \) are coefficients of a transmission and reflection of electrons at the interface between regions. And at the artificial interface one can get

\[ g^C = \cos^2(\theta/2) g^D \quad \text{at} \quad z = 0 \]

\[ g^D = \sin^2(\theta/2) g^C \quad \text{at} \quad z = 0 \]

where \( \theta \) is the angle between the magnetizations of neighboring FM films.

The total current \( I \) can obtain by following general equation

\[ I = -e\int dz f dv_s \]

where integration over \( z \) has to be carried out in each region separately, and the integration over \( v \) be performed over the spherical Fermi surface. Then the resistance of the structure can be obtained by calculating the ratio of current to the applied electric field. The relationship between resistance and applied magnetic field gives the GMR values.

As mentioned in the introduction, the transmission coefficient \( T' \) is usually treated as an invaluable value in this theoretical model. However it
is reasonable to believe that it has relationships with the thickness of nonmagnetic layer and the microstructures of the multilayer. First of all, one would expect that the expression of $T_s$ must include a Ruderman-Kittel-Kasuya-Yosida (RKKY) like oscillatory factor with the thickness of $NM$ layer. It is given by

$$K_s = \frac{k_0}{t_{NM}} \sin\left(\frac{2\pi t_{NM}}{\Lambda} + \varphi\right)$$  \hspace{1cm} (18)

where $\Lambda$ and $\varphi$ are the wavelength and phase of the oscillation, $t_{NM}$ is the thickness of $NM$ layer for separate. The parameter $k_0$ can simply set as 0.01 in the calculation.

Since the transmission coefficient represents the probability of electrons transmit form one sublayer to another, it must be affected by the interface microstructures. A good model to explain this problem is the Néel model, which describes the magnetostatic coupling caused by the correlated waviness of the magnetic layers in non-ideal interfaces. We omit the magnetization factor from the model and get

$$K_s = \frac{\pi^2 h^2}{\sqrt{2}\lambda} \exp\left(-\frac{2\pi\sqrt{2}t_{NM}}{\lambda}\right)$$  \hspace{1cm} (19)

where $h$ and $\lambda$ denote the amplitude and wavelength of the interface fluctuation respectively.

Considering that the transmission coefficient is a positive factor less than 1, we define $T'$ as

$$T' = \frac{K_1}{k_0/t_{NM}^2 + K_2} T'_{AF}$$  \hspace{1cm} (20)

where $T'_{AF}$ is the transmission coefficient in the case of antiferromagnetic coupling between the magnetization of adjacent ferromagnetic layers, which is 1 at multilayers structure.

### 2 Numerical Results and Discussions

In this section we present the results of numerical calculations for the resistance changing of Fe/Cr multilayer with our theoretical model.

Fig. 2 illustrates the magnetoresistance trace of $[\text{Fe}(2\text{ nm})/\text{Cr}(t_{Cr}\text{ nm})]_n$ multilayer calculated from the theoretical model without taking the transmission coefficient oscillation into account, i.e. $T'_{AF}=1$. In this result, the bulk SDS (spin-dependent scattering) and interface SDS are both considered by introducing the asymmetry coefficient of $FM$ layer and mixing layer, $N_b$ and $N_m$, into the calculation. Assuming there is always antiferromagnetic coupling in the multilayer, one get $T'_{AF}=1$. It is reasonable to set $R_s=1$ and $p=1$ for the multilayer structure (they are less than 1 in the sandwich structure). According to Ref.[6], we set $N_b=2.15$, $N_m=4.6$. The electron mean free paths for ferromagnetic, nonferromagnetic and mixing layer are 600, 120 and 100 nm respectively. Note that: 1) the thickness of $FM$ layer, $d_{FM}$, should take as half of the real thickness of ferromagnetic layer for one $FM$ layer belonging to two calculate units (see Fig.1). 2) The mixing layer is generally regarded as the atoms of $FM$ layer diffusing into $NM$ layer, so the real thickness of spacer is the sum of the thickness of $NM$ layer and mixing layer in calculate model. In Fig.2, the experimental results (measured at 4.5 K) are also presented[7]. We can find the calculated curve can only fit to the experimental GMR values in the antiferromagnetic coupling status.

![Fig.2](image_url)

Fig.2  Calculated GMR curve for $[\text{Fe/Cr}]_n$ multilayers as a function of $t_{Cr}$

Fig.3 presents the numerical results for the new model developed in this paper, i.e., an oscillatory transmission coefficient was introduced into the phenomenological theoretical model. The parameters to calculate $T'$ are: $\Lambda=1.1\text{ nm}$, $\varphi=0.2\pi$ in $K_1$, and $\lambda=10\text{ nm}$, $h=0.4\text{ nm}$ in $K_2$. Other parameters are same to Fig.2. A well-defined oscillation effect is seen in the calculated curve. The amplitude and period agree with the experimental data very well.
However, as we can observe in Fig.3, when $t_{Cr}$ beyond 2.5 nm ($t_{NiFe}$=2 nm) the calculated curve slightly deviates from the experimental data. The possible reason may be that the Cr layer has its own magnetism. Recent studies using neutron diffraction and Mössbauer spectroscopy show that the Néel temperature of Cr layers is much higher than room temperature [8,9]. Therefore, the contribution of the antiferromagnetism in a Cr spacer to the interlayer coupling between magnetic layers is significant. The neutron diffraction studies suggested that the magnetic structure of Cr layers separated by a ferromagnetic layer holds coherence. So when the Cr layer is thicker than magnetic layer it probably reveals the MR properties of [Cr/Fe/Cr]$_n$. One can expect that this phenomenological model will yield better simulation results if the spacer is noble metal such as Cu and Ag.

3 Conclusions

We have developed the phenomenological model in magnetic multilayer by introducing an amended transmission coefficient. We have shown that the transmission coefficient has a RKKY-like oscillatory factor, which close to the expression of interlayer exchange coupling factor in magnetic multilayer. The affections of interface roughness on the transmission coefficient have also been considered. GMR effect of [Fe(2 nm)/Cr($t_{Cr}$)]$_n$ multilayer systems is calculated with this theoretical model and found that their GMR curves agree with the experiment data well, both in the values and the oscillatory performance.

References


Brief introduction to author(s)

WEN Qiye (文岐业) was born in 1976, Ph.D student at UESTC. His research interests are magnetic films and spin-electronics.

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